Realistic Models of Low Energy Physics from Anomaly Mediated Supersymmetry Breaking

Thesis submitted in accordance with the requirements of
the University of Liverpool for the degree of Doctor in Philosophy
by

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Declaration

I hereby declare that all work described in this thesis is the result of my own research activities unless reference to others is given. None of this material has been previously submitted to this or any other university. All work was carried out in the Theoretical Physics Division of the Department of Mathematical Sciences during the period of October 2004 to September 2008.

Abstract

Anomaly Mediated Supersymmetry Breaking is an interesting possibility to explain the soft terms in the Minimal Supersymmetric Standard Model. Unfortunately, in its simplest form it does not predict the observed electroweak ground state. This work describes how, by extending the gauge symmetry of the MSSM, realistic low energy physics may be obtained from AMSB. This involves contribution to sparticle masses from a Fayet-Iliopoulos D-term. The additional factor in the gauge group is broken at high energies and decouples naturally from the low energy theory, leaving only the MSSM at a measurable scale. The mechanism by which this happens is explained and calculations of sparticle spectra are presented for the simplest case.

Unfortunately, due to a subtlety of the model, the ultraviolet insensitivity is lost. This manifests as a dynamically generated Fayet-Iliopoulos term for the abelian factor in the Standard Model gauge group. Revised mass predictions are given for this that are now realistic and can be compared with experiments in the near future.

The compatibility of this form of AMSB with $SU(5)$ Grand Unified Theories is considered. This leads to possible different phenomenology, though many of the characteristic features of the original model are preserved. Spectrum calculations for this case are given which will also be useful for comparison with experimental data.

Finally, an alternative is considered in which the gauge group is not extended and there is no FI term. This involves adding a significant contribution to one or more of the gaugino mass parameters. Two specific versions of this are discussed and spectrum calculations once again included. These spectra vary significantly from those which were calculated for the cases using the extended gauge group enough that the two possibilities could be distinguished from one another if enough data is gathered from collider experiments.

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Chapter 1

Introduction

It is a very exciting time in the history of particle physics. The Standard Model is now a well established theory which describes almost all known physics to a high degree of accuracy and it is likely that data from CERN's Large Hadron Collider experiment, or LHC, will soon provide evidence of the last particle in the model, the Higgs boson.

Even so, the Standard Model has failings, some of which are discussed in §2. There are many candidate theories as to what new physics is required to fill the holes in the standard model and fortunately there are some good reasons to believe that these may have effects in an energy range which will soon make them testable at machines such as LHC themselves. One of the best motivated ideas about physics beyond the standard model is *supersymmetry*; supersymmetry is a symmetry between matter and forces and it is both mathematically elegant and incredibly efficient at explaining away most of the Standard Model's shortcomings.

There are many specific models built using supersymmetry that may describe realistic physics. Some of these, such as those based on the idea of minimal Supergravity, or mSUGRA have been studied extensively by theorists and experimental physicists are well prepared to identify them in collider experiments.

The material in the chapters to follow deals with models which exist within a well motivated framework known as Anomaly Mediated Supersymmetry Breaking, or AMSB. When compared to mSUGRA, AMSB has received very little attention from the theory community. This is probably because the most straightforward theories involving AMSB do not reproduce the low energy physics that is observed, however there are some very realistic ways in which this problem can be avoided that produce interesting physics.

In §2 the concept of supersymmetry is explained and the Minimal Supersymmetric Standard Model is introduced as the simplest way to build an extension of the Standard Model using supersymmetry.

The material in §3 and §4 then explains how AMSB determines the parameters of the MSSM in collider-scale physics and explains how extending the gauge group may allow realistic physics to be described by AMSB, along with some naïve calculations of particle masses to demonstrate that this idea works.

§5 details why the calculations of §4 are unlikely to be accurate and how to perform realistic calculations that can be used to compare with experimental data before providing some examples of such mass predictions.

§6 explores the compatibility of the concept described in §4 and §5 with one class of Grand Unified Theories and discusses the effect on the phenomenology of the model.

In §7 an alternative mechanism to produce realistic physics from AMSB is discussed. This does not involve an extended gauge group. Calculations of masses are presented using this mechanism along with some discussion of how this scenario could be distinguished from those described in other chapters.

Throughout, the convention of natural units is employed where

$$\hbar = c = 1, \tag{1.1}$$

$\hbar$ being Planck's modified constant and c the speed of light in vacuum.

Chapter 2

The Minimal Supersymmetric Standard Model

The Minimal Supersymmetric Standard Model, commonly referred to as the MSSM, is an extension of the Standard Model of particle physics, sometimes known as the SM. It is constructed by first considering a supersymmetric equivalent to the Standard Model, and then demanding that the supersymmetry is broken by the inclusion of *soft* breaking terms, the meaning of which will be explained in §2.3. It is minimal in the sense that there is no viable supersymmetric alternative to the Standard Model with smaller field content.

The convention used will be to work in the two-component spinor formalism, since this is better suited to dealing with supersymmetry than the four-component language with which many quantum field theorists may be more familiar. A short discussion of two-component spinors can be found in §A

2.1 The Standard Model

The Standard Model of particle physics is one of the greatest achievements of twentieth century science. It is a well defined relativistic quantum field theory [1, 2] which is both elegant from a theoretical viewpoint and successful in its description of natural phenomena. It describes all observed matter and its interactions via the three forces known as the strong nuclear force, the weak nuclear force and the electromagnetic force. Two of these forces, the weak and electromagnetic are elegantly described in a unified picture reminiscent of the way in which Maxwell's classical theory of electromagnetism describes the unification of the electrostatic and magnetic forces. For a more complete review of the Standard Model see [3, 4]

2.1.1 Field Content of the SM

The SM is a gauge theory based on Quantum Chromodynamics (QCD), the theory of the strong force, and electroweak theory, the model which describes the unification of the weak and electromagnetic forces. The gauge group of QCD is $SU(3)_c$ and electroweak theory describes the breaking $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{\text{em}}$. Here, $U(1)_{\text{em}}$ is the gauge group of Quantum Electrodynamics (QED). The gauge group of the Standard Model, which shall be referred to as $\mathcal{G}_{\text{SM}}$ is therefore

$$\mathcal{G}_{\text{SM}} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y. \tag{2.1}$$

The SM also contains several fermion fields, which exist in three generations - identical except for their Yukawa couplings - and one scalar, the Higgs. The charges of the SM fields are shown in table 2.1.1. Each of these fermions exists in three generations and all the Yukawa couplings permitted by the charge assignments under $\mathcal{G}_{\text{SM}}$ are allowed.

Field	Boson	Fermion	$SU(3)$	$SU(2)$	$U(1)$
g^a	gluon		8	1	0
W^i	weak boson		1	3	0
B	hypercharge boson		1	1	0
$L = (\nu, \tau)$		Lepton doublet	1	2	-1
τ^c		Lepton singlet	1	1	2
$Q = (b, t)$		Quark doublet	3	2	1/3
t^c		Up quark singlet	$\bar{3}$	1	-4/3
b^c		Down quark singlet	$\bar{3}$	1	2/3
$H = (h^0, h^-)$	Higgs doublet		1	2	-1

Table 2.1: Field Content of the Standard Model

The Lagrangian of the Standard Model can be written

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{Higgs}}, \tag{2.2}$$

where

$$\begin{aligned}\mathcal{L}_{\text{Gauge}} &= -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{1}{4}W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &+ i\bar{L}\bar{\sigma}^\mu D_\mu L + i\bar{Q}\bar{\sigma}^\mu D_\mu Q \\ &+ i\bar{e}^c\bar{\sigma}^\mu D_\mu e^c + i\bar{u}^c\bar{\sigma}^\mu D_\mu u^c \\ &+ i\bar{d}^c\bar{\sigma}^\mu D_\mu d^c + (D_\mu H)^\dagger(D^\mu H),\end{aligned} \tag{2.3}$$

$$\mathcal{L}_{\text{Yukawa}} = QY_t\tilde{H}u^c + QY_bHd^c + LY_\tau He^c + \text{h.c}, \tag{2.4}$$

and

$$\mathcal{L}_{\text{Higgs}} = m^2H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2. \tag{2.5}$$

where the $Y_{t,b,\tau}$ are 3×3 matrices in flavour space, $\tilde{H} = i\tau^2 H^\dagger$ and $D_\mu = \partial_\mu + igA_\mu(x)$ is the covariant derivative in which g is the coupling constant and the gauge field $A_\mu(x) = \sum_a A^a_\mu(x) T^a$ is an element of the Lie algebra. The $\bar{\sigma}^\mu$ are, as described in §A,

$$\bar{\sigma}^0 = \sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{2.6}$$

$$\bar{\sigma}^1 = -\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{2.7}$$

$$\bar{\sigma}^2 = -\sigma^2 = \begin{pmatrix} 0 & \text{-i} \\ \text{i} & 0 \end{pmatrix}, \tag{2.8}$$

$$\bar{\sigma}^3 = -\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & \text{-1} \end{pmatrix}, \tag{2.9}$$

and relate to the γ-matrices used in the four-component language by

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}. \tag{2.10}$$

In this instance the Lagrangian is constructed using only one Higgs field, but it will be shown later that it is necessary in Supersymmetric theories to include at least one more.

Note that it would in principle be possible to also include a right handed neutrino, ν^c which is a singlet under $\mathcal{G}_{SM}$ in order to include a Yukawa mass for the neutrino. While the experimental data is now overwhelmingly in favour of neutrino masses they are constrained to be so much smaller than the vacuum expectation value of the Higgs field that including them in this way immediately poses the question of why this should be.

2.1.2 Electroweak Symmetry Breaking

Masses in the Standard Model are generated by the spontaneous breaking of the symmetry $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{\text{em}}$, during which process the Higgs field obtains a non-zero vacuum expectation value [5, 6, 7]. Three of the four real degrees of freedom belonging to the complex doublet Higgs field (the would-be Goldstone bosons) become the longitudinal polarisations of the weak bosons, making them massive and leaving a single, real scalar as the observable Higgs.

The mass eigenstates in the broken theory are the two charged W-bosons, the Z and the massless photon. In terms of the fields in the unbroken theory these are

$$W^\pm_\mu = \frac{W^1_\mu \mp W^2_\mu}{\sqrt{2}}, \tag{2.11}$$

$$Z_\mu = -B_\mu \sin\theta_W + W^3_\mu \cos\theta_W, \tag{2.12}$$

$$\gamma_\mu = B_\mu \cos\theta_W + W^3_\mu \sin\theta_W. \tag{2.13}$$

where θ_W is the weak mixing angle.

The masses of the W and Z bosons are

$$m_W = \frac{1}{\sqrt{2}} g v, \qquad m_Z = \frac{m_W}{\cos\theta_W}, \tag{2.14}$$

where v is the vacuum expectation value of the Higgs. The empirical value for $\sin\theta_W$ at m_Z is 0.23120.

2.1.3 Problems with the Standard Model

In spite of its successes, the Standard Model is an incomplete description of nature.

Perhaps the most obvious failing is the omission of any description of gravity, which is still best understood in the language of general relativity. For the purpose of most calculations this need not cause a problem since any corrections by gravity would be negligible at the scale of SM physics.

It was long believed that neutrinos are massless, yet recent evidence now makes this almost impossible to believe. It is likely that neutrinos acquire such a small mass through the see-saw mechanism [8] whereby a gauge-singlet, 'right-handed neutrino', ν^c has a large Dirac mass (this is perfectly natural since this mass is not set by the size of the Higgs vacuum expectation value) and the Lagrangian contains a mass term involving both the left and right-handed neutrinos such as $\mathcal{L} \supset L Y_\nu \tilde{H} \nu^c$. The masses are then the eigenvalues of the matrix

$$m_\nu = \begin{pmatrix} 0 & \mathcal{O}(M_Z) \\ \mathcal{O}(M_Z) & M_{\text{heavy}} \end{pmatrix}, \tag{2.15}$$

where $M_{\text{heavy}} \gg M_Z$. This then has one eigenvalue $\mathcal{O}(M_{\text{heavy}})$ and one $\mathcal{O}(\frac{M_Z^2}{M_{\text{heavy}}})$ with very little mixing between the gauge eigenstates.

Modern cosmological wisdom tells us that most of the matter in the universe is comprised of so-called Dark Matter, which does not scatter photons. Though objects such as black holes would contribute to the total Dark Matter of the universe it is generally believed that the greatest contribution comes from non-baryonic matter. The only candidate Dark Matter particle in the SM is the neutrino but the data is not consistent with more than a small contribution from neutrinos.

Additionally, the strong force is not readily included in the mechanism which so nicely unifies the weak and electromagnetic forces. This is not in itself a problem, but, aside from being aesthetically desirable, when one looks at the running values of the coupling constants the value for the strong force passes close enough to the point at which the other two meet as to suggest possible new physics which may allow such a unification.

Perhaps the main reason cited as motivation for theories of physics beyond the Standard Model is the Hierarchy Problem. Why is there such a huge difference between the scale of SM physics ($m_W = 80.4$GeV) and the Planck mass ($M_{Pl} = \mathcal{O}(10^{19}\text{GeV})$) which one would expect to be the most natural scale of physics? This will now be briefly discussed.

The Hierarchy Problem

The Hierarchy Problem is more than a concern about the aesthetics of the Standard Model. There are contributions to the Higgs mass which scale quadratically to any ultraviolet momentum cutoff Λ_{UV}. If one wishes Λ_{UV} to be small then one must also consider what new physics enters at this scale. One such contribution, from a spin 1/2 fermion-loop is shown in figure (2.1).

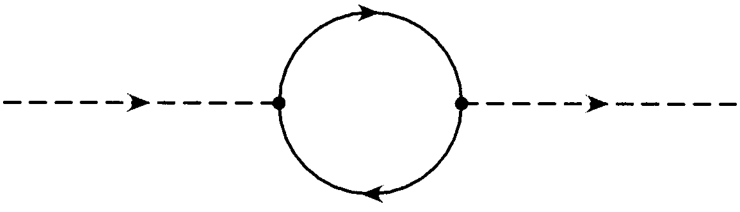

Figure 2.1: Quadratic contribution to the Higgs $(\text{mass})^2$ from a fermion-loop

The most popular solution to the Hierarchy Problem is supersymmetry. This is a proposed symmetry between fermions and bosons. Its most important consequence being that for every field there should exist a *superpartner* which has exactly the same quantities except that its spin should differ from the original by 1/2. A more complete, mathematical explanation of this will be attempted in the sections to follow, but it is possible to understand how it acts as a cure to the Hierarchy problem without need for a deep understanding.

If a supersymmetry is imposed on the standard model then the fermion in the above diagram would have a superpartner known as the *sfermion* which has exactly the same properties as the original fermion except that it is a scalar. In addition to figure (2.1) there is now a diagram involving a sfermion-loop, as shown in figure (2.2.

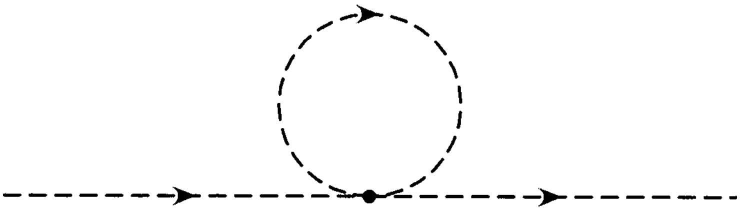

Figure 2.2: Quadratic contribution to the Higgs $(\text{mass})^2$ from a sfermion-loop

This contribution exactly cancels the one from figure (2.1) because of the different sign associated with fermion and boson loops.

2.2 Supersymmetry

This section will provide a brief introduction to Supersymmetry. For a full review see [9, 10].

Supersymmetry, as has already been stated, is a symmetry between bosons and fermions. The operator which transforms a bosonic state into a fermionic state, Q is an anticommuting spinor such that

$$Q|\text{Fermion}\rangle = |\text{Boson}\rangle, \qquad Q|\text{Boson}\rangle = |\text{Fermion}\rangle \tag{2.16}$$

The hermitian conjugate of Q, $\bar{Q}$ is also a supersymmetry generator, and since they carry spin 1/2 supersymmetry is a symmetry of spacetime. It is possible to consider cases where there are multiple pairs of supersymmetry generators Q and $\bar{Q}$ but in this document there is always $\mathcal{N} = 1$ supersymmetry where only a single pair exists.

The group of isometries of Minkowsky space is called the Poincaré group whose generators are the translations, P_μ, and Lorentz transformations $M^{\mu\nu}$. Its algebra is

$$[P_\mu, P_\nu] = 0, \tag{2.17}$$

$$[P_\mu, M_{\rho\sigma}] = i(\eta_{\mu\rho}P_\sigma - \eta_{\mu\sigma}P_\rho), \tag{2.18}$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = i(\eta_{\nu\rho}M_{\mu\sigma} - \eta_{\nu\sigma}M_{\mu\rho} - \eta_{\mu\rho}M_{\nu\sigma} + \eta_{\mu\sigma}M_{\nu\rho}) \tag{2.19}$$

where $\eta_{\mu\nu}$ is the metric of Minkowsky space

$$(\eta_{\mu\nu}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{2.20}$$

The Coleman-Mandula theorem of [11] states that this is the only possible bosonic spacetime symmetry group of a consistent relativistic quantum field theory on Minkowsky space. It is, however, possible to include *fermionic* generators [12] and extend the Poincaré algebra to a *graded* Lie algebra that will also contain anticommutators. The only way this can be done is via supersymmetry [13] so supersymmetry is the only spacetime symmetry that is not included in the standard model.

2.2.1 The Supersymmetry Algebra

The supersymmetry algebra for one supersymmetry is

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2(\sigma^\mu)_{\alpha\dot{\alpha}}P_\mu, \tag{2.21}$$

$$[Q_\alpha, P_\mu] = 0, \tag{2.22}$$

$$[Q_\alpha, M_{\mu\nu}] = \frac{1}{2}(\sigma_{\mu\nu})_\alpha^\beta Q_\beta, \tag{2.23}$$

$$[\bar{Q}_{\dot{\alpha}}, M_{\mu\nu}] = -\frac{1}{2}\bar{Q}_{\dot{\beta}}(\bar{\sigma}_{\mu\nu})^{\dot{\beta}}_{\dot{\alpha}}, \tag{2.24}$$

$$\{Q_\alpha, Q_\beta\} = 0, \tag{2.25}$$

$$\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0 \tag{2.26}$$

where

$$\sigma^{\mu\nu} = \frac{1}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu), \tag{2.27}$$

$$\bar{\sigma}^{\mu\nu} = \frac{1}{4}(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\nu). \tag{2.28}$$

2.2.2 The Wess-Zumino Model

The simplest supersymmetric quantum field theory was first described by Wess and Zumino [14]. It describes a massless complex scalar field ϕ and its fermion superpartner ψ. This is called a *chiral supermultiplet.* The simplest action which can be written down involving these fields contains just the kinetic terms for each

$$S = \int d^4x(\mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{fermion}}), \tag{2.29}$$

where

$$\mathcal{L}_{\text{scalar}} = -\partial^\mu\phi^*\partial_\mu\phi, \tag{2.30}$$

$$\mathcal{L}_{\text{fermion}} = -i\psi^\dagger\bar{\sigma}^\mu\partial_\mu\psi. \tag{2.31}$$

A supersymmetry transformation should turn the scalar field into something involving the fermion field. The simplest possibility is

$$\delta\phi = \epsilon\psi, \tag{2.32}$$

with

$$\delta\phi^* = \epsilon^\dagger\psi^\dagger, \tag{2.33}$$

where ϵ^α is an infinitesimal, anticommuting Weyl fermion object. If the spinor field transforms

$$\delta\psi_\alpha = i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi, \tag{2.34}$$

$$\delta\psi^\dagger_{\dot{\alpha}} = -i(\epsilon\sigma^\mu)_{\dot{\alpha}}\partial_\mu\phi^*, \tag{2.35}$$

then the Lagrangian is invariant under the transformations *on-shell*, i.e. when the classical equations of motion hold. In order to make the Lagrangian invariant quantum mechanically it is necessary to introduce a complex scalar field F with dimensions of $(\text{mass})^2$ called an auxiliary field,

$$\mathcal{L}_{\text{auxiliary}} = -F^*F. \tag{2.36}$$

Notice that F has no propagator. Introducing F also balances the number of fermionic and bosonic degrees of freedom. That these should be the same is a fundamental prediction of supersymmetry.

The action

$$S_{\text{Wess-Zumino}} = \int d^4x(\mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{auxiliary}}), \tag{2.37}$$

is invariant under the transformations

$$\begin{aligned}
\delta\phi &= \epsilon\psi, && (2.38)\\
\delta\phi^* &= \epsilon^\dagger\psi^\dagger, && (2.39)\\
\delta\psi_\alpha &= i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi + \epsilon_\alpha F, && (2.40)\\
\delta\psi^\dagger_{\dot\alpha} &= -i(\epsilon\sigma^\mu)_{\dot\alpha}\partial_\mu\phi^* + \epsilon^\dagger_{\dot\alpha}F^*, && (2.41)\\
\delta F &= i\epsilon^\dagger\bar\sigma^\mu\partial_\mu\psi, && (2.42)\\
\delta F^* &= -i\partial_\mu\psi^\dagger\bar\sigma^\mu\epsilon. && (2.43)
\end{aligned}$$

In generalising this theory to one which contains several bosonic and fermionic fields, ϕ_i, ψ_i, and where interactions between these fields are included, it can be shown that the only allowed interaction terms for a renormalisable theory are described by

$$\mathcal{L}_{\text{int}} = -\frac{1}{2}\frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\psi_i\psi_j + \frac{\partial W}{\partial\phi_i}F_i + \text{h.c.}, \tag{2.44}$$

where W is called the *superpotential*. For a renormalisable theory the superpotential takes the form

$$W = \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}h^{ijk}\phi_i\phi_j\phi_k. \tag{2.45}$$

Usually, the auxiliary fields F_i are eliminated by their equations of motion

$$F_i = -W_i^*, \tag{2.46}$$

so that

$$\mathcal{L}_{\text{auxiliary}} = W^iW_i^*. \tag{2.47}$$

2.2.3 Supersymmetry and Gauge Fields

A similar treatment of a theory containing a gauge field A^a_μ and its superpartner, a two-component Weyl fermion *gaugino* λ^a (which together form a *gauge supermultiplet*) yields the Lagrangian

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} - i\lambda^\dagger\bar\sigma^\mu D_\mu\lambda^a + \frac{1}{2}D^aD^a, \tag{2.48}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu \tag{2.49}$$

is the usual Yang-Mills field strength, and

$$D_\mu\lambda^a = \partial_\mu\lambda^a + gf^{abc}A^b_\mu\lambda^c \tag{2.50}$$

is the covariant derivative of λ^a. The field D^a is an auxiliary field with dimensions of $(\text{mass})^2$ and fills a similar role to F. Only one bosonic degree of freedom is needed here to balance the fermionic degrees of freedom so D^a is constrained to be real.

The gauge coupling g is a function of the energy scale Q due to renormalisation. This scale dependence is described by the gauge β-function $\beta_g = Q\frac{\partial}{\partial Q}g$.

This lagrangian is invariant under the supersymmetry transformations

$$\delta A^a_\mu = \frac{1}{\sqrt{2}}\left(\epsilon^\dagger\bar{\sigma}_\mu\lambda^a + \lambda^{\dagger a}\bar{\sigma}_\mu\epsilon\right), \tag{2.51}$$

$$\delta\lambda^a_\alpha = \frac{i}{2\sqrt{2}}(\sigma^\mu\bar{\sigma}^\nu\epsilon)_\alpha F^a_{\mu\nu} + \frac{1}{\sqrt{2}}\epsilon_\alpha D^a, \tag{2.52}$$

$$\delta D^a = \frac{i}{\sqrt{2}}\left(\epsilon^\dagger\bar{\sigma}^\mu D_\mu\lambda^a - D_\mu\lambda^{\dagger a}\bar{\sigma}^\mu\epsilon\right). \tag{2.53}$$

For a model containing both chiral and gauge supermultiplets, the lagrangian, involving some additional interaction terms, is

$$\mathcal{L} = \mathcal{L}_{\text{chiral}} + \mathcal{L}_{\text{gauge}} \tag{2.54}$$

$$-\sqrt{2}g(\phi^*T^a\psi)\lambda^a - \sqrt{2}g\lambda^{\dagger a}(\psi^\dagger T^a\phi) + g(\phi^*T^a\phi)D^a. \tag{2.55}$$

The auxiliary field D^a is usually replaced using its equation of motion

$$D^a = -g(\phi^*T^a\phi). \tag{2.56}$$

If the gauge group is abelian it is possible to include an additional term that is linear in the auxiliary field D. This is known as a *Fayet-Iliopoulos* term,

$$\mathcal{L}_{FI} = \xi D, \tag{2.57}$$

and is both supersymmetric and gauge invariant by itself. This will be important in later chapters.

2.2.4 SUSY and GUT

As was mentioned briefly in §2.1.3, the running of the coupling constants in the Standard Model suggests a possible unification of QCD with electroweak theory. The coupling constants do not converge at a single value but pass close enough to each other that many people believe in new physics that would allow such a unification. This new physics would need to exist at some scale between the low energies which have already been observed and the scale at which any higher symmetry breaks to $\mathcal{G}_{\text{SM}}$.

While the hierarchy problem is probably the most commonly cited reason why people believe there may be supersymmetry, many people who seek to construct Grand Unified Theories are encouraged that supersymmetry, a framework well motivated outside their area, allows coupling constant unification. The running of the coupling constants is the main reason for this, since even the most minimal form of supersymmetry that is in agreement with current observations (see §2.4) allows the coupling constants to meet at a single value without the need for additional field content.

In addition to this being a good reason for protagonists of GUTs to believe in supersymmetry, it provides a strong argument that anyone who wishes to build a model involving supersymmetry might also wish to consider how their model might be compatible with existing GUT theories.

2.3 Supersymmetry Breaking

Supersymmetry clearly predicts a large number of particles that have never been observed in any conclusive experiment. It predicts that these particles should have the same properties as existing ones except for their spins, including that they should have the same mass. This is impossible since such particles would be produced in huge numbers and would have a massive impact on everyday physics.

From this one should deduce, with absolute certainty, that one of the following statements is true.

There is no supersymmetry in nature.

Supersymmetry exists as a broken symmetry where particles do not have the same mass as their superpartners.

As has been highlighted, there are many reasons to believe that there is supersymmetry in nature; the elegant way in which supersymmetry extends the Poincaré algebra, the solution to the hierarchy problem and the compatibility of supersymmetry and GUTs are compelling reasons to study supersymmetry even if it must be a broken symmetry.

In order for supersymmetry to solve the hierarchy problem without an unacceptable level of fine-tuning, it is necessary that the superpartners exist at a relatively light scale, and that the manner in which it is broken does not introduce additional quadratic divergences. Supersymmetry breaking that does not introduce quadratic divergences is known as *soft*.

The most general soft breaking terms in the Lagrangian of a holomorphic theory are

$$\mathcal{L}_{\text{soft}} = -\left(\frac{1}{2}M_a\lambda^a\lambda^a + \frac{1}{6}h^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + t^i\phi_i\right) + \text{h.c.} - (m^2)^i_j\phi^{j*}\phi_i. \quad (2.58)$$

Of course, in realistic models where these fields are the superpartners of observed fields many of these terms are ruled out, for instance the t^i must all be zero for any field that is not a singlet.

There are several candidate theories as to what the precise mechanism of supersymmetry breaking may be and how SUSY breaking might be communicated to the visible sector. The most studied scenarios relate to minimal Supergravity, or mSUGRA, where sets of the soft parameters are assumed to be equal to one another at some high scale.

This document deals with the low energy consequences of an alternative mechanism that is well motivated but not as well studied called Anomaly Mediated Supersymmetry Breaking, AMSB.

One might wonder as to why virtually all of the particles in the standard model should have been observed without any evidence that anyone has seen their superpartners. If the energy range of experiment is such that we have been able to see half of the particles that exist in nature then is it reasonable to think we might only have seen *one* field from *every* supermultiplet? The answer to this is that those particles that have been observed are the ones whose masses are set by the scale of the Higgs vacuum expectation value (or, like the photon, they are massless), so they should be expected to be found in one range of energies, while the remaining particles have masses set by whatever scale is determined by the mechanism of supersymmetry breaking. It is of course possible that this scale is much higher than one can hope to observe with collider experiments, but if it is to be hoped that supersymmetry provides the answer to the hierarchy problem then there is good reason to believe that new physics may be visible at the next generation of colliders.

For a more detailed review see [15].

2.4 MSSM

The Minimal Supersymmetric Standard Model, commonly known as the MSSM, is the simplest way to extend the Standard Model into a realistic theory with broken supersymmetry. The spin 1/2 quarks and leptons have spin 0 superpartners which are given the names *squarks* and *sleptons* (though 's' is often given to the specific name for the sake of brevity, e.g. the top squark is sometimes called simply a stop). The spin 1 gauge bosons have spin 1/2 superpartners called *gauginos* (again the '-ino' is often applied to the name of the specific field e.g. gluino, Wino). By convention the superpartners of the SM fields are denoted by the same symbols but with a tilde over the top.

The Higgs is slightly more difficult to include in the MSSM, since one cannot use $i\tau^2 H^\dagger$ to give mass to the up-type quarks as in the SM but instead must use a second Higgs doublet. This is also important because with only one Higgs doublet there is a gauge anomaly which can easily be avoided by including a second doublet with opposite charge under $U(1)_Y$. The superpartners of the Higgs fields are called 'Higgsinos'.

If one tries to write down all the terms that would be allowed in the MSSM with arbitrary soft SUSY breaking, the Lagrangian would violate conservation of baryon number and lepton number such as LQd^c. To avoid this, most models include the requirement that the MSSM conserves R-parity, which is defined as

$$P_R = (-1)^{3(B-L)+2s}, \tag{2.59}$$

where B is baryon number, L is lepton number and s is the spin. All the SM particles have $P_R = +1$, while all their superpartners have $P_R = -1$. An interesting side effect of this constraint is that the lightest particle with $P_R = -1$, known as the lightest supersymmetric particle or LSP, is stable. If the LSP is electrically neutral it can be an excellent candidate for Dark Matter, which may explain another of the Standard Model's shortcomings - for a review see [16]. Including R-Parity also means that sparticles (those particles with $P_R = -1$) can only be produced in even numbers.

After electroweak symmetry breaking, the neutral gauginos and neutral Higgsinos mix to form four observable states known as *neutralinos*, generally denoted χ_i^0 while the charged gauginos and charged Higgsinos mix to form four (two positive and two negative) *charginos*, $\chi_i^{\pm}$. The lightest neutralino is often the LSP and is a very good Dark Matter candidate because it has only weak interactions.

There are five observable Higgs bosons after electroweak symmetry breaking. These are two CP-even neutral particles, h and H, a CP-odd neutral particle, A and two charged particles $H^{\pm}$.

The superpotential of the MSSM is

$$W_{MSSM} = QY_u t^c H_2 + QY_d b^c H_1 + LY_e \tau^c H_1 + \mu H_1 H_2. \tag{2.60}$$

Chapter 3

Anomaly Mediated Supersymmetry Breaking

In §4, four sets of renormalisation group invariant equations describing all the soft terms of the MSSM will be introduced. The ultraviolet insensitivity alone would be an interesting reason to study the sparticle spectra predicted by these equations, but remarkably they also happen to be the same relations for the soft terms predicted in [17].

This chapter contains a brief discussion of a mechanism called *Anomaly Mediated Supersymmetry Breaking* as proposed in [17] as further motivation for the physics to follow in later chapters.

Many models in modern theoretical physics, such string theories, require that there are more than the usual $(3+1)$ dimensions of spacetime. In the case to be considered here, the visible sector fields - i.e. Standard Model fields - are restricted to a four dimensional subspace (a 3-*brane*) in the higher dimensions. Supersymmetry breaking can then occur on another brane, or through a nonlocal effect in the full higher dimensional space.

When considering the four-dimensional effective theory, it is useful to define the chiral superfield,

$$\Phi = 1 + F_\Phi \theta^2, \tag{3.1}$$

where F_Φ is the spin-0 auxiliary field of the supergravity multiplet. It should be understood that Φ here is nothing more than a tool for separating out the F_Φ.

The general Lagrangian in the low-energy theory can be written up to two-derivative order as

$$\begin{aligned} \mathcal{L} &= \sqrt{-g}\Big(\int d^4\theta f(Q^\dagger, e^{-V}Q)\Phi^\dagger\Phi + \int d^2\theta(\Phi^3 W(Q) + \tau(Q)\mathcal{W}_\alpha^2) + \text{ h.c.} \\ &- \frac{1}{6} f(\tilde{q}^\dagger, \tilde{q})(R + \text{ vector auxiliary terms } + \text{ gravitino terms}, \end{aligned} \tag{3.2}$$

where Q are the chiral matter superfields, $\tilde{q}$ are their lowest bosonic components, V

are the vector superfields, $\mathcal{W}_\alpha$ are their supersymmetric field strengths and

$$f = -3M_{Pl}^2 e^{-K/3M_{Pl}^2}, \tag{3.3}$$

where M_{Pl} is the Planck mass and K is the supergravity Kähler potential.

It appears from this that the visible sector Lagrangian does not couple to Φ. This corresponds to a *classical* invariance of the visible sector Lagrangian under super-Weyl transformations where Φ is multiplied by an arbitrary chiral superfield. This allows one to completely Weyl-transform $\langle\Phi\rangle$ away from the supergravity multiplet which couples to the Yang-Mills theory. However, the super-Weyl transformation is anomalous, resulting in

$$\tau \to \tau - 2b_0 \ln(\Phi), \tag{3.4}$$

where b_0 is the one-loop β-function coefficient from the anomaly.

Upon evaluation of the θ integral, there is a gaugino mass term

$$M = -b_0 g^2 F_\Phi \tag{3.5}$$

where g is the gauge coupling.

This demonstrates how the soft terms may have their origin in Anomaly Mediated Supersymmetry Breaking. The result here occurs at one-loop and will dominate only if there are no higher order contributions. Calculation of the relations for all the soft terms in AMSB gives the same results as presented in §4.

Chapter 4

Renormalisation group Invariant Soft Terms and Fayet-Iliopoulos D-terms

The Anomaly Mediation mechanism of supersymmetry breaking, which has been discussed in detail in [17] and [18] and which has been mentioned briefly in §3, is a well defined framework within which all the soft terms of the theory are determined by a single mass parameter, $m_{3/2}$.

The soft terms are defined by the following four sets of equations (in which the summation convention is not used):

$$M_i = m_{3/2}\frac{\beta_{g_i}}{g_i}, \tag{4.1}$$

$$h^{ijk} = -m_{3/2}\beta_Y^{ijk}, \tag{4.2}$$

$$(m^2)^i{}_j = \frac{1}{2}m_{3/2}^2 Q\frac{d}{dQ}\gamma^i{}_j, \tag{4.3}$$

$$b^{ij} = \kappa m_{3/2}\mu^{ij} - m_{3/2}\beta_\mu^{ij}. \tag{4.4}$$

Here β_{g_i} are the gauge β-functions, γ the chiral supermultiplet anomalous dimensions and β_Y^{ijk} the Yukawa β-functions for Yukawa couplings of the form $Y^{ijk}\phi_i\phi_j\phi_k$ and Q is the energy scale. For $M_{SUSY} \approx 1$TeV we require $m_{3/2} \approx 40$TeV.

In this form, Anomaly Mediated Supersymmetry Breaking (AMSB) has a number of very attractive features. There are very few parameters, these equations are renormalisation group invariant [19] and the framework is very predictive.

As an example to illustrate the renormalisation group invariance of this set of equations, consider (4.1). Acting with the operator $Q\frac{\partial}{\partial Q}$ on the left gives

$$\begin{aligned}\beta_{M_i} &= m_{3/2}Q\frac{\partial}{\partial Q}\left(\frac{\beta_{g_i}}{g_i}\right),\\ &= m_{3/2}\left(\beta_{g_i}\frac{\partial}{\partial g_i} + \beta_Y^{klm}\frac{\partial}{\partial Y^{klm}} + \beta_Y^{klm*}\frac{\partial}{\partial Y^{klm*}}\right)\left(\frac{\beta_{g_i}}{g_i}\right).\end{aligned} \tag{4.5}$$

It is known from [19, 20, 21, 22, 23] that

$$\beta_{M_i} = 2\left(M_i g_j^2 \frac{\partial}{\partial g_j^2} - h^{klm}\frac{\partial}{\partial Y^{klm}}\right)\left(\frac{\beta_{g_i}}{g_i}\right). \tag{4.6}$$

The first part of this is clearly the same since

$$g_j^2\frac{\partial}{\partial g_j^2} = \frac{g_j}{2}\frac{\partial}{\partial g_j}, \tag{4.7}$$

and substituting (4.1) for the M_i gives

$$\begin{aligned} M_i g_j^2\frac{\partial}{\partial g_j^2} &= m_{3/2}\frac{\beta_{g_i}}{g_i}\frac{g_j}{2}\frac{\partial}{\partial g_j}, \\ &= m_{3/2}\frac{1}{2}\beta_{g_i}\frac{\partial}{\partial g_i}. \end{aligned} \tag{4.8}$$

Comparing this with (4.6) and (4.5), it can be seen that (4.1) is renormalisation group invariant if

$$m_{3/2}\left(\beta_Y^{klm}\frac{\partial}{\partial Y^{klm}} + \beta_Y^{klm*}\frac{\partial}{\partial Y^{klm*}}\right) = -2h^{klm}\frac{\partial}{\partial Y^{klm}}. \tag{4.9}$$

To see that this is so, one must consider (4.2) and substitute this into the right hand side of (4.9):

$$-2h^{klm}\frac{\partial}{\partial Y^{klm}} = 2m_{3/2}\beta_Y^{klm}\frac{\partial}{\partial Y^{klm}}. \tag{4.10}$$

(4.6) is now

$$\beta_{M_i} = m_{3/2}\left(\beta_{g_k}\frac{\partial}{\partial g_k} + 2\beta_Y^{klm}\frac{\partial}{\partial Y^{klm}}\right)\left(\frac{\beta_{g_i}}{g_i}\right), \tag{4.11}$$

which is the same as (4.5) if

$$\beta_Y^{klm}\frac{\partial}{\partial Y^{klm}}\left(\frac{\beta_{g_i}}{g_i}\right) = \beta_Y^{klm*}\frac{\partial}{\partial Y^{klm*}}\left(\frac{\beta_{g_i}}{g_i}\right), \tag{4.12}$$

which is shown to be so in the appendix of [19].

Unfortunately, any attempt to perform phenomenology with AMSB in the MSSM at this point will fail.

To understand this, consider the set of equations (4.3), which gives the masses of the scalar components of the chiral supermultiplets. This can also be written:

$$(m^2)^i{}_j = \frac{1}{2}m_{3/2}^2\left(\beta_Y^{klm}\frac{d}{dY^{klm}}\gamma^i{}_j + \beta_Y^{klm*}\frac{d}{dY^{klm*}}\gamma^i{}_j + \beta_{g_k}\frac{d}{dg_k}\gamma^i{}_j\right), \tag{4.13}$$

where there is understood to be no summation over the i and j indices. Since most of the Yukawa couplings are small, the anomalous dimensions are usually dominated by the part dependent on the gauge coupling. The leading contributions to the scalar $(\text{mass})^2$ are therefore roughly proportional to $-g_i\beta_{g_i}$.

For the squarks this does not present a problem, since the β-function for QCD is negative so the overall $(\text{mass})^2$ is positive. But any chiral supermultiplet field which does not couple to a gauge group with an asymptotically free coupling constant would have a scalar component with a negative (mass)^2. The sleptons clearly suffer from this problem, which means that it would be impossible to obtain the usual electroweak vacuum state which breaks $SU(2)_L \otimes U(1)_Y \to U(1)_{\text{em}}$.

Many spectrum calculations (for example those performed using SOFTSUSY [24]) which have been produced for AMSB have attempted to circumvent this problem by including an additional term with dimensions of (mass)^2 in the set of equations (4.3) that is common to all the scalars. This produces the physical mass $\tilde{m}$, defined $(\tilde{m}^2)^i{}_j = (m^2)^i{}_j + m_0^2$.

Unfortunately, while this does allow for calculation of a realistic spectrum, it is undesirable for two reasons. The first of these is that it is completely lacking in motivation, and the second is the more serious fact that it destroys the renormalisation group invariance that was such an attractive feature of the original scenario.

4.1 Inclusion of the Fayet-Iliopoulos D-term

It was observed in [25] that the renormalisation group invariance is preserved if (4.3) is replaced by

$$(\tilde{m}^2)^i{}_j = \frac{1}{2} m_{3/2}^2 Q \frac{d}{dQ} \gamma^i{}_j + \xi (\mathcal{Y}')^i{}_j, \tag{4.14}$$

where ξ is a constant and $\mathcal{Y}'$ is a matrix whose ${}^i{}_j$th entry corresponds is the hypercharge corresponding to the $\phi^i \phi^j$ coupling to an anomaly free $U(1)$, $U(1)'$. The additional term here corresponds to a Fayet-Iliopoulos D-term as described in equation (2.57).

It is possible to construct a very natural model which leads us to this solution. This model was first described in [27] and will be repeated in detail here. In the model, the gauge group is $\mathcal{G}_{\text{SM}} \otimes U(1)'$. The Lagrangian contains a large ($\gg M_{\text{SUSY}}$) Fayet-Iliopoulos D-term for the $U(1)'$, $\xi D'$. The $U(1)'$ charges of the MSSM fields and new fields introduced in this model will be discussed shortly.

4.1.1 Anomaly-free $U(1)'$

It is possible to define an anomaly-free, flavour-blind $U(1)$ within the MSSM whose charges are totally constrained by just two degrees of freedom. Since the purpose of introducing this $U(1)'$ is to give a positive Fayet-Iliopoulos contribution to the sleptons, it is sensible to choose these degrees of freedom to be the $U(1)'$ charges of the lepton doublet, L, and singlet, e. To calculate the charges of the other fields, one must consider the triangle diagrams contributing to the $U(1)'^3$ anomaly and those triangle diagrams involving mixed anomalies with $\mathcal{G}_{\text{SM}}$. These mixed anomalies are $U(1)_Y^2 U(1)'$, $U(1)_Y U(1)'^2$, $SU(2)_L^2 U(1)'$ and $SU(3)_c^2 U(1)'$.

$$A(U(1)'^3) = 2L^3 + e^3 + 6Q^3 + 3u^{c3} + 3d^{c3} + 2H_1^3 + 2H_2^3 + \nu^{c3}, \tag{4.15}$$

$$A(U(1)_Y^2U(1)') = 2L + 4e + \frac{2}{3}Q + \frac{16}{3}u^c + \frac{4}{3}d^c + 2H_1 + 2H_2, \quad (4.16)$$

$$A(U(1)_YU(1)'^2) = -2L^2 + 2c^2 + 2Q^2 - 4u^{c2} + 2d^{c2} - 2H_1 + 2H_2, \quad (4.17)$$

$$A(SU(2)_L^2U(1)') = 4L + 12Q + 4H_1 + 4H_2, \quad (4.18)$$

$$A(SU(3)_c^2U(1)') = 18Q + 9u^c + 9d^c, \quad (4.19)$$

$$A(U(1)' - grav) = 2L + e + 36Q + 3u^c + 3d^c + 2H_1 + 2H_2 + \nu^c, \quad (4.20)$$

where the symbols representing the MSSM fields (including the right handed neutrino ν^c) are understood to represent their $U(1)'$ charges in this case.

All these anomalies vanish when the charge assignments of table 4.1.1 are substituted into equations (4.15-4.19).

Q	u^c	d^c	H_1	H_2	ν^c
$-\frac{1}{3}L$	$-e-\frac{2}{3}L$	$e+\frac{4}{3}L$	$-e-L$	$e+L$	$-2L-e$

Table 4.1: Charges for an anomaly-free $U(1)'$

4.2 Spontaneously Broken $U(1)'$

Consider the superpotential $W = W_1 + W_2$, where W_1 is the usual superpotential of the MSSM as described in §2 and W_2 contains fields which are singlets under $\mathcal{G}_{\rm SM}$,

$$W_2 = \lambda_1\phi\bar{\phi}s + \lambda_2\phi\nu^c\nu^c, \quad (4.21)$$

where ϕ and $\bar{\phi}$ carry opposite $U(1)'$ charges, $\pm q_\phi$, ν^c has $U(1)'$ charge $-\frac{1}{2}q_\phi$ and s is a singlet under the entire gauge group $\mathcal{G}_{\rm SM}\otimes U(1)'$.

Consider the following part of the scalar potential involving the fields ϕ and $\bar{\phi}$:

$$V_D = m_\phi^2\phi^*\phi + m_{\bar{\phi}}^2\bar{\phi}^*\bar{\phi} - \frac{1}{2}\left[\xi + q_\phi(\phi^*\phi - \bar{\phi}^*\bar{\phi}) + \sum_i e_i\chi_i^*\chi_i\right]^2, \quad (4.22)$$

where the masses m_ϕ^2 and $m_{\bar{\phi}}^2$ are generated according to the usual AMSB relation (4.3) and the other term is the $U(1)'$ D-term (including the Fayet-Iliopoulos parameter ξ). The χ_i are all the scalar fields of the MSSM and e_i are their $U(1)'$ charges. Let it be assumed that the F-term of W_2 does not affect the minimisation of this potential.

To find the minimum of this potential one must first find its derivatives

$$\frac{\partial V_D}{\partial\phi} = m_\phi^2\phi^* - q_\phi\phi^*\left[\xi + q_\phi(\phi^*\phi - \bar{\phi}^*\bar{\phi}) + \sum_i e_i\chi_i^*\chi_i\right], \quad (4.23)$$

$$\frac{\partial V_D}{\partial\bar{\phi}} = m_{\bar{\phi}}^2\bar{\phi}^* + q_\phi\bar{\phi}^*\left[\xi + q_\phi(\phi^*\phi - \bar{\phi}^*\bar{\phi}) + \sum_i e_i\chi_i^*\chi_i\right]. \quad (4.24)$$

Seeking a minimum where $\bar{\phi} = 0$, one obtains

$$\begin{aligned} \langle \phi^* \phi \rangle &= \frac{q_\phi \xi - m_\phi^2}{q_\phi^2} \\ &\equiv \frac{1}{2} v_\phi^2, \end{aligned} \tag{4.25}$$

so $\langle \phi \rangle \approx \mathcal{O}(\sqrt{\xi})$. ξ is chosen to be large so that $\langle \phi \rangle \gg M_{SUSY}$.

By writing $\phi = \langle (v_\phi + H(x)) \rangle / \sqrt{2}$ it is straightforward to expand V_D about this minimum,

$$\begin{aligned} V_D &= \frac{m_\phi^2 \xi}{q_\phi} - \frac{m_\phi^4}{2q_\phi^2} + \left(m_\phi^2 + m_{\bar{\phi}}^2 + \frac{1}{2} v_\phi^2 \lambda_1^2 \right) \bar{\phi}^* \bar{\phi} - \sum_i e_i \frac{m_\phi^2}{q_\phi} \chi_i^* \chi_i \\ &+ \frac{1}{2} v_\phi^2 \lambda_1^2 s^* s + \frac{1}{2} v_\phi^2 (\nu^c)^* \nu^c + \frac{1}{2} \left(v_\phi q_\phi H - q_\phi \bar{\phi}^* \bar{\phi} + \sum_i e_i \chi_i^* \chi_i \right)^2 + \ldots \end{aligned} \tag{4.26}$$

At this point, one should notice that the only contributions to the masses of the MSSM fields are $\mathcal{O}(M_{SUSY})$. All the $\mathcal{O}(\sqrt{\xi})$ contributions to these masses have cancelled without any fine tuning or tweaking of the parameters. It is now sensible to define the *effective* Fayet-Iliopoulos parameter $\xi' \equiv m_\phi^2 / q_\phi$. It is this ξ' rather than the ξ present in the Lagrangian which should appear in the set of equations (4.14).

Consider also the following tree level interactions:

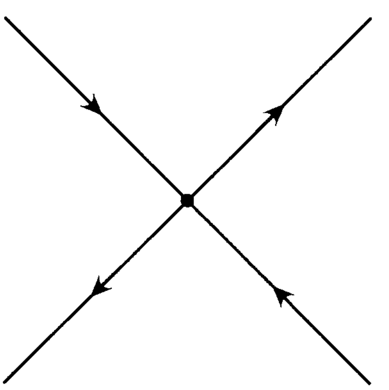

Figure 4.1: $(\chi_i^* \chi_i)^2$ vertex

Figure (4.1) is the additional interaction between the scalar component of each chiral supermultiplet in the theory arising from the term $\frac{1}{2}(\sum_i e_i \chi_i^* \chi_i)^2$ in the potential. Each external leg is a χ_i and the interaction is proportional to $2c_i^2$. Figure (4.2) has the same external legs but now includes the exchange of the heavy scalar $H(x)$. This interaction is proportional to $-q_\phi v_\phi c_i$.

At first thought it appears that these will have consequences for the low energy theory, consider the interaction involving the addition of these two diagrams (call it I).

$$I \propto 2c_i^2 + 2(-q_\phi v_\phi c_i)^2 \frac{1}{k^2 - q_\phi^2 v_\phi^2} \tag{4.27}$$

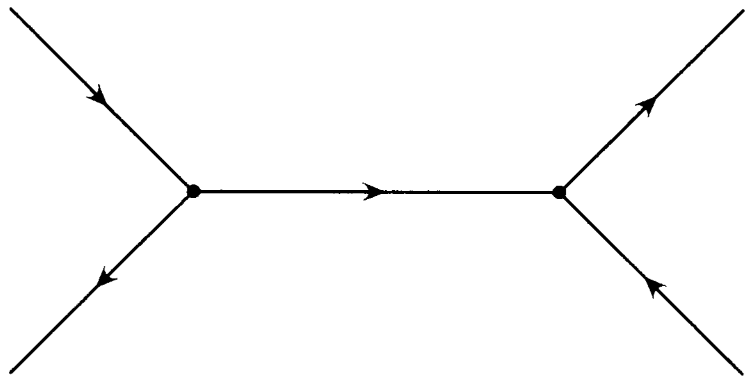

Figure 4.2: Contribution to $(\chi_i^*\chi_i)^2$ vertex from $\chi_i^*\chi_i H$

where k is the momentum in the internal leg and the factor of two in the second term appears because by interchanging the external momenta in Figure (4.2) one can construct two equivalent contributions. In the low energy limit of everyday physics, $k^2 \ll q_\phi^2 v_\phi^2$ which gives

$$I \to 2e_i^2 + 2q_\phi^2 v_\phi^2 e_i^2 \frac{1}{-q_\phi^2 v_\phi^2} = 0. \tag{4.28}$$

This neat demonstration of the decoupling in the low energy limit has all, once again occurred without any interference from the model builder and is as natural a mechanism in this model as it is an elegant one. In the low energy theory there is only the MSSM where the soft terms are given by the Anomaly Mediation conditions with (4.3) replaced by (4.14).

Notice here that the large vacuum expectation value for ϕ gives a large mass to ν^c thereby naturally implementing the see-saw mechanism.

4.3 Calculation of Precision Sparticle Spectra

Throughout these calculations the approximation

$$Y_t = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \lambda_t \end{pmatrix}, \tag{4.29}$$

$$Y_b = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \lambda_b \end{pmatrix}, \tag{4.30}$$

$$Y_\tau = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \lambda_\tau \end{pmatrix} \tag{4.31}$$

was used. Since this leads to a degeneracy in the first two generations the second generation is omitted from the following results tables.

Due to the decoupling of the $U(1)'$ in the low energy limit (as described in the previous section). The anomalous dimensions of the fields are unchanged from those of the MSSM. For the third generations at one loop these are:

$$16\pi^2\gamma_{H_1} = 3\lambda_b^2 + \lambda_\tau^2 - \frac{3}{2}g_2^2 - \frac{3}{10}g_1^2, \tag{4.32}$$

$$16\pi^2\gamma_{H_2} = 3\lambda_t^2 - \frac{3}{2}g_2^2 - \frac{3}{10}g_1^2, \tag{4.33}$$

$$16\pi^2\gamma_L = \lambda_\tau^2 - \frac{3}{2}g_2^2 - \frac{3}{10}g_1^2, \tag{4.34}$$

$$16\pi^2\gamma_Q = \lambda_b^2 + \lambda_t^2 - \frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{1}{30}g_1^2, \tag{4.35}$$

$$16\pi^2\gamma_{t^c} = 2\lambda_t^2 - \frac{8}{3}g_3^2 - \frac{8}{15}g_1^2, \tag{4.36}$$

$$16\pi^2\gamma_{b^c} = 2\lambda_b^2 - \frac{8}{3}g_3^2 - \frac{2}{15}g_1^2, \tag{4.37}$$

$$16\pi^2\gamma_{\tau^c} = 2\lambda_\tau^2 - \frac{6}{5}g_1^2, \tag{4.38}$$

The expressions for the first two generations are the same but without the Yukawa couplings.

The soft scalar masses are

$$\bar{m}_Q^2 = m_Q^2 - \frac{1}{3}L\xi', \tag{4.39}$$

$$\bar{m}_{t^c}^2 = m_{t^c}^2 - \left(\frac{2}{3}L + e\right)\xi', \tag{4.40}$$

$$\bar{m}_{b^c}^2 = m_{b^c}^2 + \left(\frac{4}{3}L + e\right)\xi', \tag{4.41}$$

$$\bar{m}_L^2 = m_L^2 + L\xi', \tag{4.42}$$

$$\bar{m}_{\tau^c}^3 = m_{\tau^c}^2 + e\xi', \tag{4.43}$$

$$\bar{m}_{H_1}^2 = m_{H_1}^2 - (e+L)\xi', \tag{4.44}$$

$$\bar{m}_{H_2}^2 = m_{H_2}^2 + (e+L)\xi', \tag{4.45}$$

where

$$m_Q^2 = \frac{1}{2}m_{3/2}^2 Q\frac{d}{dQ}\gamma_Q \tag{4.46}$$

and so on.

The third generation A-parameters are

$$A_t = -m_{3/2}(\gamma_Q + \gamma_{t^c} + \gamma_{H_2}), \tag{4.47}$$

$$A_b = -m_{3/2}(\gamma_Q + \gamma_{b^c} + \gamma_{H_1}), \tag{4.48}$$

$$A_\tau = -m_{3/2}(\gamma_L + \gamma_{\tau^c} + \gamma_{H_1}) \tag{4.49}$$

and the corresponding first and second generation quantities have been set to zero. The gaugino masses are

$$M_i = m_{3/2}\left|\frac{\beta_g}{g}\right|. \tag{4.50}$$

The scale of the Fayet-Iliopoulos contributions to the scalar masses is set by the AMSB contribution to m_ϕ^2 so, as has already been discussed, is naturally expected to be the same order as the other AMSB contributions. These contributions depend on two parameters, $L\xi'$ and $e\xi'$. It is possible, and desirable for the sake of simplicity, to set $\xi' = 1(\mathrm{Tev})^2$ by redefinition of both L and e. This is done from now on.

Spectrum calculations begin by choosing input values for $m_{3/2}$, $\tan\beta$, L, e and the sign of μ. These are then used to calculate appropriate dimensionless coupling input values at M_Z by iteration and loop corrections as described in [28]. Sparticle pole masses are calculated by running the dimensionless couplings up to this mass and using equations (4.39-4.50) with the full one-loop corrections from [28].

The effect of using one, two and three-loop anomalous dimensions and β-functions is compared as in [29]. It should be noted that the three loop calculation included the full three-loop approximation for both β-functions *and* anomalous dimensions so some higher order effects are included through products of these functions.

Choosing $m_{3/2} \approx 40\mathrm{TeV}$ produces sparticle masses that are interesting for low energy physics.

The calculations were performed using Maple and a sample of the code is included in the appendix §C.

4.4 Sparticle Spectra

Figure (4.3) shows the region of (e, L) charges which produces an acceptable spectrum. The bounding line which is crossed when either e or L becomes to large is defined by the experimental limit on the CP-odd Higgs mass while the curve preventing e and L from taking values that are too small is defined by the experimental limit on the lightest sneutrino and charged slepton.

Notice in the spectra that follow that the lightest Higgs lies below the experimental bound for the standard model case. Whether this is ruled out requires a more detailed analysis, but it should be remembered that this chapter has been compiled as a demonstration that the general mechanism works and allows calculations. In the more realistic models and analyses that follow in later chapters the entire spectrum falls comfortably within the experimentally accepted range. It is also worth commenting that these spectrum calculations were originally performed at a time when the top was believed to be significantly heavier and that the light Higgs in this case would have been just above the SM bound. Since the value for the top mass still carries a fairly large error it is entirely possible that further measurements of this quantity may yield a result which would push this value for the Higgs mass up again.

The tables show spectra for several different choices of the input parameters. In each case the LSP is marked by an asterisk by its three-loop mass.

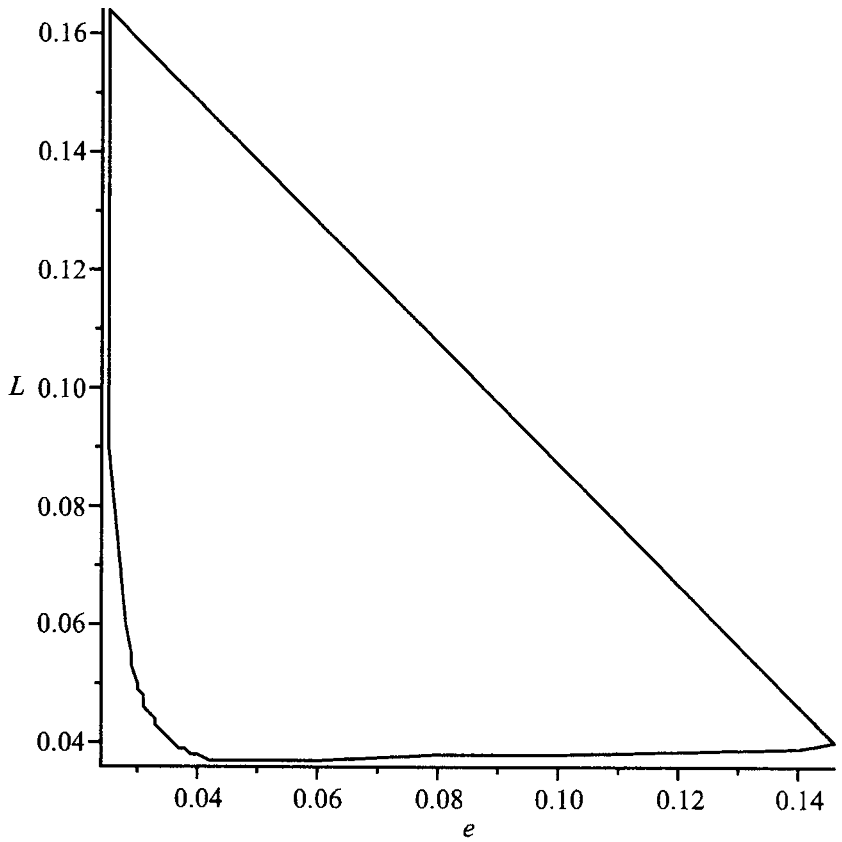

Figure 4.3: Allowed region of (e, L) for $m_{3/2} = 40\text{TeV}$, $\tan\beta = 10$ and $\mu > 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	914	890	887
$\tilde{t}_1$	501	497	484
$\tilde{t}_2$	764	755	745
$\tilde{b}_1$	938	929	919
$\tilde{b}_2$	718	709	698
$\tilde{\tau}_1$	253	250	250
$\tilde{\tau}_2$	208	204	204
$\tilde{u}_R$	769	759	746
$\tilde{u}_L$	827	812	801
$\tilde{d}_R$	950	941	932
$\tilde{d}_L$	830	816	805
χ_1^0	111	135	136*
χ_2^0	356	364	364
χ_3^0	559	580	572
χ_4^0	564	585	577
$\chi_1^{\pm}$	112	136	136
$\chi_2^{\pm}$	564	585	577
h	112	112	112
H	313	347	333
A	314	347	333
$H^{\pm}$	324	356	343
$\tilde{e}_R$	228	228	228
$\tilde{e}_L$	241	234	235
$\tilde{\nu}_\tau$	225	218	218
$\tilde{\nu}_e$	227	220	220

Table 4.2: Spectrum for $m_{3/2} = 40$TeV, $L = 0.08$, $e = 0.07$, $\tan\beta = 10$ and $\mu < 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	914	890	887
$\tilde{t}_1$	512	509	497
$\tilde{t}_2$	766	756	747
$\tilde{b}_1$	929	920	910
$\tilde{b}_2$	731	721	711
$\tilde{\tau}_1$	284	284	284
$\tilde{\tau}_2$	123	108	109
$\tilde{u}_R$	767	756	744
$\tilde{u}_L$	835	820	810
$\tilde{d}_R$	937	929	919
$\tilde{d}_L$	838	824	813
χ_1^0	106	130	130
χ_2^0	354	362	362
χ_3^0	568	589	582
χ_4^0	579	600	593
$\chi_1^\pm$	106	130	130
$\chi_2^\pm$	576	597	590
h	109	109	109
H	363	393	381
A	363	393	381
$H^\pm$	372	401	390
$\tilde{e}_R$	286	285	285
$\tilde{e}_L$	132	118	118
$\tilde{\nu}_\tau$	99	79	80*
$\tilde{\nu}_e$	104	85	86

Table 4.3: Spectrum for $m_{3/2} = 40$TeV, $L = 1/25$, $c = 1/10$, $\tan\beta = 10$ and $\mu > 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	914	890	887
$\tilde{t}_1$	509	506	494
$\tilde{t}_2$	757	748	738
$\tilde{b}_1$	908	900	890
$\tilde{b}_2$	717	708	698
$\tilde{\tau}_1$	283	283	283
$\tilde{\tau}_2$	95	73	75
$\tilde{u}_R$	767	757	744
$\tilde{u}_L$	834	821	810
$\tilde{d}_R$	937	929	919
$\tilde{d}_L$	838	824	814
χ_1^0	107	131	131
χ_2^0	355	363	363
χ_3^0	563	584	576
χ_4^0	572	594	586
$\chi_1^\pm$	107	131	131
$\chi_2^\pm$	571	591	584
h	111	111	111
H	257	295	280
A	258	296	281
$H^\pm$	271	307	292
$\tilde{e}_R$	285	285	285
$\tilde{e}_L$	132	118	119
$\tilde{\nu}_\tau$	85	59	60*
$\tilde{\nu}_e$	104	85	86

Table 4.4: Spectrum for $m_{3/2} = 40$TeV, $L = 1/25$, $c = 1/10$, $\tan\beta = 20$ and $\mu > 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	914	890	887
$\tilde{t}_1$	536	532	520
$\tilde{t}_2$	766	755	745
$\tilde{b}_1$	922	913	903
$\tilde{b}_2$	727	717	707
$\tilde{\tau}_1$	219	216	216
$\tilde{\tau}_2$	171	164	164
$\tilde{u}_R$	784	774	762
$\tilde{u}_L$	830	816	805
$\tilde{d}_R$	930	922	912
$\tilde{d}_L$	834	820	809
χ_1^0	106	130	131*
χ_2^0	354	362	362
χ_3^0	586	606	599
χ_4^0	596	616	609
$\chi_1^\pm$	107	131	131
$\chi_2^\pm$	594	614	607
h	109	109	109
H	417	443	432
A	417	442	432
$H^\pm$	425	450	439
$\tilde{e}_R$	204	204	204
$\tilde{e}_L$	195	187	187
$\tilde{\nu}_\tau$	175	166	166
$\tilde{\nu}_e$	178	167	169

Table 4.5: Spectrum for $m_{3/2} = 40$TeV, $L = 0.06$, $e = 0.06$, $\tan\beta = 10$ and $\mu > 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	914	890	887
$\tilde{t}_1$	509	506	493
$\tilde{t}_2$	754	745	734
$\tilde{b}_1$	949	941	932
$\tilde{b}_2$	716	707	696
$\tilde{\tau}_1$	281	276	276
$\tilde{\tau}_2$	197	195	196
$\tilde{u}_R$	767	756	744
$\tilde{u}_L$	822	808	797
$\tilde{d}_R$	958	950	941
$\tilde{d}_L$	826	812	801
χ_1^0	106	130	130*
χ_2^0	354	362	361
χ_3^0	550	571	563
χ_4^0	561	583	575
$\chi_1^\pm$	106	130	131
$\chi_2^\pm$	558	580	572
h	109	109	109
H	298	335	321
A	297	334	320
$H^\pm$	308	344	331
$\tilde{e}_R$	206	206	206
$\tilde{e}_L$	280	274	274
$\tilde{\nu}_\tau$	266	260	260
$\tilde{\nu}_e$	268	262	262

Table 4.6: Spectrum for $m_{3/2} = 40$TeV, $L = 0.1$, $e = 0.06$, $\tan\beta = 10$ and $\mu > 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	914	890	887
$\tilde{t}_1$	494	491	478
$\tilde{t}_2$	749	739	728
$\tilde{b}_1$	963	955	946
$\tilde{b}_2$	710	701	690
$\tilde{\tau}_1$	314	309	309
$\tilde{\tau}_2$	199	199	198
$\tilde{u}_R$	758	747	735
$\tilde{u}_L$	818	804	793
$\tilde{d}_R$	972	964	955
$\tilde{d}_L$	822	808	797
χ_1^0	106	130	130*
χ_2^0	353	361	361
χ_3^0	531	553	545
χ_4^0	543	565	558
$\chi_1^\pm$	106	130	130
$\chi_2^\pm$	539	562	554
h	109	109	109
H	210	264	246
A	209	264	245
$H^\pm$	226	276	258
$\tilde{e}_R$	207	207	206
$\tilde{e}_L$	314	308	309
$\tilde{\nu}_\tau$	301	296	296
$\tilde{\nu}_e$	303	298	298

Table 4.7: Spectrum for $m_{3/2} = 40$TeV, $L = 0.12$, $e = 0.06$, $\tan\beta = 10$ and $\mu > 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	914	890	887
$\tilde{t}_1$	499	496	483
$\tilde{t}_2$	745	735	725
$\tilde{b}_1$	967	958	949
$\tilde{b}_2$	705	696	685
$\tilde{\tau}_1$	344	339	339
$\tilde{\tau}_2$	144	143	143
$\tilde{u}_R$	762	752	739
$\tilde{u}_L$	814	800	788
$\tilde{d}_R$	976	967	958
$\tilde{d}_L$	818	804	792
χ_1^0	106	130	130*
χ_2^0	354	362	361
χ_3^0	531	553	545
χ_4^0	543	566	558
$\chi_1^\pm$	106	130	130
$\chi_2^\pm$	540	562	554
h	109	109	109
H	211	265	246
A	210	264	245
$H^\pm$	226	276	259
$\tilde{e}_R$	153	152	152
$\tilde{e}_L$	344	339	340
$\tilde{\nu}_\tau$	333	328	328
$\tilde{\nu}_e$	335	330	330

Table 4.8: Spectrum for $m_{3/2} = 40\text{TeV}$, $L = 0.14$, $e = 0.04$, $\tan\beta = 10$ and $\mu > 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	913	889	887
$\tilde{t}_1$	492	489	476
$\tilde{t}_2$	743	733	723
$\tilde{b}_1$	974	965	956
$\tilde{b}_2$	703	693	682
$\tilde{\tau}_1$	358	353	354
$\tilde{\tau}_2$	145	144	144
$\tilde{u}_R$	758	747	734
$\tilde{u}_L$	812	798	786
$\tilde{d}_R$	982	974	965
$\tilde{d}_L$	816	801	790
χ_1^0	106	130	130*
χ_2^0	354	361	361
χ_3^0	521	544	536
χ_4^0	534	557	549
$\chi_1^\pm$	106	130	130
$\chi_2^\pm$	530	553	545
h	108	109	109
H	140	220	196
A	129	219	195
$H^\pm$	163	234	212
$\tilde{e}_R$	153	153	152
$\tilde{e}_L$	358	354	354
$\tilde{\nu}_\tau$	348	343	343
$\tilde{\nu}_e$	349	345	345

Table 4.9: Spectrum for $m_{3/2} = 40$TeV, $L = 0.15$, $e = 0.04$, $\tan\beta = 10$ and $\mu > 0$

4.5 A Scale Free Model

An attractive feature of AMSB is that the scale M_{SUSY} is determined entirely by $m_{3/2}$. The model described in the earlier part of §4 depends also on two other apparently arbitrarily determined scales, these are the Higgs μ-term and the Fayet-Iliopoulos term ξ (although dependence on the latter is not as severe a problem as it first seems since it does not appear in the expressions for the scalar masses). A model has been proposed [30] that is similar in motivation yet whose Lagrangian does not explicitly contain either of these two scales. This model will be outlined here along with some results showing that at one-loop the physics is virtually identical to that of the previous model, as was of course the intention of this revised version.

4.5.1 The Superpotential

The superpotential for this theory is

$$W = W_1 + W_2 + W_3, \tag{4.51}$$

where W_1 contains Yukawa terms involving fields charged under $\mathcal{G}_{\text{SM}}$ and the right handed neutrino ν^c,

$$W_1 = QY_u t^c H_2 + QY_d b^c H_1 + LY_e \tau^c H_1 + LY_\nu \nu^c H_2 \tag{4.52}$$

and W_2 contains terms involving fields which are singlets under $\mathcal{G}_{\text{SM}}$, a pair of fields ϕ and $\bar{\phi}$ which are charged under $U(1)'$ and two gauge singlets, Z and U,

$$W_2 = U\left(\lambda\phi\bar{\phi} - \frac{1}{2}\rho Z^2\right) + \frac{1}{6}kU^3. \tag{4.53}$$

W_3 contains terms involving both sectors and is used to generate the Higgs μ-term and the right handed neutrino masses. There are two slightly different forms for W_3, each of which resembles a more common form of non-minimal Supersymmetric model. These are

$$W_3^A = \lambda'' U H_1 H_2 + \frac{1}{2}Y_{\nu^c}\phi\nu^c\nu^c, \tag{4.54}$$

$$W_3^B = \lambda'' Z H_1 H_2 + \frac{1}{2}Y_{\nu^c}\phi\nu^c\nu^c. \tag{4.55}$$

In this superpotential, λ, ρ, k and λ'' are coupling constants while Y_{u,d,e,ν,ν^c} are 3×3 matrices in flavour space. This superpotential is natural in that it contains all cubic terms allowed under the symmetry $\mathcal{G}_{\text{SM}} \otimes U(1)' \otimes Z_2$ where under the Z_2, $Z \to -Z$ and if $W_3 = W_3^A$ the remaining fields are invariant while if $W_3 = W_3^B$ they are arranged so that $H_1H_2 \to -H_1H_2$ while preserving the invariance of W_1.

Both of these possibilities involve non-minimal supersymmetry, and at least to a superficial degree they resemble well studied scenarios. For $W_3 = W_3^A$ the mechanism is much the same as in the next-to-Minimal Supersymmetric Standard Model (nMSSM) [31] though some of the low energy physics differs since the light singlet state decouples. For $W_3 = W_3^B$ the superpotential looks very like that of the Minimal-non-minimal Supersymmetric Standard Model (MNSSM) [32], however the MNSSM requires the addition of certain discrete symmetries in order to protect against dangerous tadpoles of the singlet at high orders. These tadpoles are present in this theory but are suppressed by multiple powers of λ'' which, in this case, is required to be small.

At this point one should of course notice that the superpotential, W, contains no explicit mass scale. The electroweak scale and the scale of supersymmetry breaking are generated entirely by AMSB, while the scale at which the $U(1)'$ is broken is determined by a very elegant mechanism which shall now be discussed.

4.5.2 The Scalar Potential

The soft terms of this model take the general form

$$\begin{aligned}
\mathcal{L}_{\text{Soft}} &= \sum_{\chi} m_\chi^2 \chi^* \chi + \left[m_3^2 H_1 H_2 + \sum_{i=1}^{3} M_i \lambda_i \lambda_i + h.c. \right] \\
&+ \left[H_2 Q h_u t^c + H_1 Q h_d b^c + H_1 L h_e \tau^c + h.c. \right], \qquad (4.56)
\end{aligned}$$

where M_i, h_Y and m_χ^2 are given by the usual AMSB relations (4.1-4.3). Begin by writing down the scalar potential corresponding to W_2 and seek an extremum such that ϕ, $\bar{\phi}$ and Z obtain vacuum expectation values of the same order $\langle Z \rangle \gg m_{3/2}$. The potential is

$$\begin{aligned}
V &= \frac{1}{2} m_1^2 x^2 + \frac{1}{2} m_2^2 y^2 + \frac{1}{2} m_3^2 z^2 + \frac{1}{2} m_u^2 u^2 + h_1 uxy + \frac{1}{2} h2uz^2 + \frac{1}{6} h_3 u^3 \\
&+ \frac{1}{4} \lambda^2 \left(xy - z^2 + \frac{1}{2} \bar{k} u^2 \right)^2 + \frac{1}{4} \lambda^2 u^2 (x^2 + y^2) + \frac{1}{2} \lambda \rho u^2 z^2 \\
&+ \frac{1}{8} g_\phi (x^2 - y^2)^2, \qquad (4.57)
\end{aligned}$$

where $g_\phi = q_\phi g'$, g' is the $U(1)'$ coupling constant $x/\sqrt{2} = \langle \phi \rangle$, $y/\sqrt{2} = \langle \bar{\phi} \rangle$, $z = \sqrt{\frac{\rho}{\lambda}} \langle Z \rangle$, $u/\sqrt{2} = \langle U \rangle$, $m_1 = m_\phi$, $m_2 = m_{\bar{\phi}}$, $m_3^2 = 2 m_Z^2 \lambda/\rho$, $\bar{k} = k/\lambda$, $h_1 = h_\lambda/\sqrt{2}$, $h_2 = -\sqrt{2}\frac{\lambda}{\rho} h_\rho$ and $h_3 = h_k/\sqrt{2}$. h_λ, h_ρ and h_k are the trilinear soft parameters corresponding to couplings between the same fields as λ, ρ and k respectively and are determined by equation (4.2).

At an extremum

$$x \left[m_1^2 + \frac{1}{2} g_\phi^2 (x^2 - y^2) + \frac{1}{2} \lambda^2 u^2 \right] + y \left[\frac{1}{2} \lambda^2 \left(xy - z^2 + \frac{\bar{k}}{2} u^2 \right) + h_1 u \right] = 0, \qquad (4.58)$$

$$y \left[m_2^2 - \frac{1}{2} g_\phi^2 (x^2 - y^2) + \frac{1}{2} \lambda^2 u^2 \right] + x \left[\frac{1}{2} \lambda^2 \left(xy - z^2 + \frac{\bar{k}}{2} u^2 \right) + h_1 u \right] = 0, \qquad (4.59)$$

$$z \left[m_3^2 - \lambda^2 \left(xy - z^2 + \frac{\bar{k}}{2} u^2 \right) + \lambda \rho u^2 + h_2 u \right] = 0, \qquad (4.60)$$

$$u\left[m_u^2+\frac{1}{2}\lambda^2(x^2+y^2)+\frac{\bar{k}}{2}\lambda^2\left(xy-z^2+\frac{\bar{k}}{2}u^2\right)+\lambda\rho z^2\right]$$
$$+h_1xy+\frac{1}{2}h_2z^2+\frac{1}{2}h_3u^2 \quad = \quad 0. \tag{4.61}$$

For the desired extremum to be a minimum it is required that $x \sim y \sim z \gg u$. Setting $x \approx y \approx z \approx M$, (4.61) then gives

$$u \approx \frac{2h_1+h_2}{2(\lambda^2+\lambda\rho)} \tag{4.62}$$

which is naturally $\mathcal{O}\left(\frac{m_{3/2}}{16\pi^2}\right)$. Since AMSB normally requires $m_{3/2}$ to be $\mathcal{O}$(40TeV) for TeV scale sparticle spectrum this gives, when $W_3 = W_3^A$, a Higgs μ-term of the right magnitude without having to assume a small value for λ''. For $W_3 = W_3^B$, λ'' must be small. To leading order, (4.58) and (4.59) then give

$$x^2-y^2=\frac{m_2^2-m_1^2}{g_\phi^2} \tag{4.63}$$

which gives a contribution to the scalar masses in a similar manner to that discussed earlier where, for example, the contribution to the slepton doublet mass is

$$\Delta m_L^2 = \frac{1}{2}g_\phi^2\frac{q_L}{q_\phi}(x^2-y^2)=\frac{m_2^2-m_1^2}{2}\frac{q_L}{q_\phi}. \tag{4.64}$$

Since m_1 and m_2 depend on unknown couplings, it is economical to write

$$\xi \equiv \frac{m_2^2-m_1^2}{2q_\phi} \tag{4.65}$$

and ξ will henceforth play precisely the role ξ' played in the later stages of the scale-dependent model.

Setting $x = R\cos\Omega$ and $y = R\sin\Omega$ leads to

$$\sin 2\Omega = -\frac{m_3^2+(2h_1+h_2)u+\lambda\rho u^2}{m_1^2+m_2^2+\lambda^2u^2}. \tag{4.66}$$

Since the minimum is at $x \sim y$ this corresponds to $\Omega \approx \pi/4$ which, remembering (4.62) gives

$$\Delta \equiv m_1^2+m_2^2+m_3^2-\frac{(h_1+\frac{1}{2}h_2)^2}{\lambda(\lambda+\rho)}=0+\mathcal{O}\left(\frac{m_{3/2}^4}{M^2}\right). \tag{4.67}$$

At first sight this may look like a hideous fine-tuning. However, since all these parameters are functions of scale, and this is considering the region at which they take a similar value, the best scale at which to consider these parameters is at a scale equal to their own scale; this minimises the effect of radiative corrections.

From (4.57), it is easy to see that for $m_1^2+m_2^2+m_3^2<0$, V is unbounded from below. However, since $m_1^2+m_2^2+m_3^2$ is a function of scale it is quite natural to have $\Delta<0$ at some scale, say $Q<X$ and $\Delta>0$ at a higher scale $Q>X$. One can therefore conclude that V has a minimum at a scale where (4.67) is satisfied without any fine tuning whatsoever. This mechanism is an example of dimensional transmutation [33].

4.5.3 Calculation of Precision Sparticle Spectra

At one-loop with $W_3 = W_3^A$ the anomalous dimensions for the fields which are $\mathcal{G}_{\rm SM}$ singlets are

$$
\begin{aligned}
16\pi^2\gamma_\phi &= \lambda^2 + \frac{1}{2}\mathrm{Tr}(Y_{\nu^c})^2 - 2g'^2q_\phi^2, && (4.68)\\
16\pi^2\gamma_{\bar\phi} &= \lambda^2 - 2g'^2q_\phi^2, && (4.69)\\
16\pi^2\gamma_U &= \lambda^2 + \frac{1}{2}\rho^2 + \frac{1}{2}k^2 + 2\lambda''^2, && (4.70)\\
16\pi^2\gamma_Z &= \rho^2. && (4.71)
\end{aligned}
$$

For $W_3 = W_3^B$ the expressions for γ_ϕ and $\gamma_{\bar\phi}$ are the same but

$$
\begin{aligned}
16\pi^2\gamma_U &= \lambda^2 + \frac{1}{2}\rho^2 + \frac{1}{2}k^2, && (4.72)\\
16\pi^2\gamma_Z &= \rho^2 + 2\lambda''^2. && (4.73)
\end{aligned}
$$

For the MSSM fields the anomalous dimensions are

$$
\begin{aligned}
16\pi^2\gamma_{H_1} &= 3\lambda_b^2 + \lambda_\tau^2 - \frac{3}{2}g_2^2 - \frac{3}{10}g_1^2 + \lambda''^2 - 2C_H, && (4.74)\\
16\pi^2\gamma_{H_2} &= 3\lambda_t^2 - \frac{3}{2}g_2^2 - \frac{3}{10}g_1^2 + \lambda''^2 - 2C_H, && (4.75)\\
16\pi^2\gamma_L &= \lambda_\tau^2 - \frac{3}{2}g_2^2 - \frac{3}{10}g_1^2 - 2C_L, && (4.76)\\
16\pi^2\gamma_Q &= \lambda_b^2 + \lambda_t^2 - \frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{1}{30}g_1^2 - 2C_Q, && (4.77)\\
16\pi^2\gamma_{t^c} &= 2\lambda_t^2 - \frac{8}{3}g_3^2 - \frac{8}{15}g_1^2 - 2C_{t^c}, && (4.78)\\
16\pi^2\gamma_{b^c} &= 2\lambda_b^2 - \frac{8}{3}g_3^2 - \frac{2}{15}g_1^2 - 2C_{b^c}, && (4.79)\\
16\pi^2\gamma_{\tau^c} &= 2\lambda_\tau^2 - \frac{6}{5}g_1^2 - 2C_{\tau^c}, && (4.80)
\end{aligned}
$$

where

$$
\begin{aligned}
C_H &= q_H^2 g'^2, && (4.81)\\
C_L &= q_L^2 g'^2, && (4.82)\\
C_Q &= q_Q^2 g'^2, && (4.83)\\
C_{t^c} &= q_{t^c}^2 g'^2, && (4.84)\\
C_{b^c} &= q_{b^c}^2 g'^2, && (4.85)\\
C_{\tau^c} &= q_{\tau^c}^2 g'^2. && (4.86)
\end{aligned}
$$

with $q_H^2 = q_{H_1}^2 = q_{H_2}^2$ since $q_{H_1} = -q_{H_2}$.

The expressions for (4.39-4.50) remain the same.

4.5.4 Sparticle Spectrum

The spectrum presented here is to one-loop for both the scale-dependent scenario and the scale free alternative described throughout this section. They are virtually identical which demonstrates how successfully the original model which depended on explicit scales has been rewritten as an equivalent model in which all scales are generated naturally.

Mass/GeV	Explicit μ and ξ	Scale Free
$\tilde{g}$	914	914
$\tilde{t}_1$	501	501
$\tilde{t}_2$	755	757
$\tilde{b}_1$	941	941
$\tilde{b}_2$	717	717
$\tilde{\tau}_1$	263	263
$\tilde{\tau}_2$	215	215
$\tilde{u}_R$	762	762
$\tilde{u}_L$	827	826
$\tilde{d}_R$	955	955
$\tilde{d}_L$	830	830
χ_1^0	107	107
χ_2^0	354	354
χ_3^0	549	549
χ_4^0	560	560
$\chi_1^\pm$	107	107
$\chi_2^\pm$	557	557
h	110	112
H	271	271
A	271	270
$H^\pm$	283	283
$\tilde{e}_R$	249	249
$\tilde{e}_L$	241	241
$\tilde{\nu}_\tau$	223	223
$\tilde{\nu}_e$	227	227

Table 4.10: Comparrison of spectra in the two cases for $m_{3/2} = 40$TeV, $L = 0.08$, $e = 0.08$, $\tan\beta = 13$, $g' = g_2$ and $\mu > 0$

Chapter 5

Renormalisation Group Effects of Multiple Fayet-Iliopoulos Terms

In everything that has been discussed so far it has been assumed that the ultraviolet insensitivity of AMSB is preserved when (4.3) is replaced by (4.14). There is however a subtlety regarding the renormalisation group invariance, concerning the $U(1)$ of $\mathcal{G}_{\rm SM}$. Suppose that the FI term associated with the SM $U(1)$ is zero at the high scale at which the $U(1)'$ is broken, M_X. A Fayet-Iliopoulos term for the SM $U(1)$ would be dynamically generated through interaction with the $U(1)'$ FI term. This effect is enough to become significant at low energies.

It is possible [34] to reparameterise these two Fayet-Iliopoulos contributions as a single term in the low energy theory, but this term will be dependent on the scale. This point is easily shown; if one considers the situation for two Fayet-Iliopoulos terms, ξ_1 and ξ_2, with lepton doublet and singlet charges L_1, e_1 and L_2, e_2 respectively then (4.14) could be rewritten as

$$\bar{m}_L^2 = m_L^2 + L_1\xi_1 + L_2\xi_2 \equiv m_L^2 + L''\xi'', \tag{5.1}$$

$$\bar{m}_{\tau^c}^2 = m_{\tau^c}^2 + e_1\xi_1 + e_2\xi_2 \equiv m_{\tau^c}^2 + e''\xi''. \tag{5.2}$$

It follows from here that

$$\bar{m}_Q^2 = m_Q^2 - \frac{1}{3}L_1\xi_1 - \frac{1}{3}L_2\xi_2 = m_Q^2 - \frac{1}{3}L''\xi'', \tag{5.3}$$

$$\bar{m}_{t^c}^2 = m_{t^c}^2 - \left(\frac{2}{3}L_1 + e_1\right)\xi_1 - \left(\frac{2}{3}L_2 + e_2\right)\xi_2 = m_{t^c}^2 - \left(\frac{2}{3}L'' + e''\right)\xi'', \tag{5.4}$$

$$\bar{m}_{b^c}^2 = m_{b^c}^2 + \left(\frac{4}{3}L_1 + e_1\right)\xi_1 + \left(\frac{4}{3}L_2 + e_2\right)\xi_2 = m_{b^c}^2 + \left(\frac{4}{3}L'' + e''\right)\xi'', \tag{5.5}$$

$$\bar{m}_{H_1}^2 = m_{H_1}^2 - (e_1 + L_1)\xi_1 - (e_2 + L_2)\xi_2 = m_{H_1}^2 - (e'' + L'')\xi'', \tag{5.6}$$

$$\bar{m}_{H_2}^2 = m_{H_2}^2 + (e_1 + L_1)\xi_1 + (e_2 + L_2)\xi_2 = m_{H_2}^2 + (e'' + L'')\xi''. \tag{5.7}$$

It is of course possible to make the assumption that the FI term for the SM $U(1)$ is zero at the scale $M_{\rm SUSY}$, in which case the previous calculations have merely neglected the relatively small effects of running around this scale. However, since there is no reason to assume any relationship between the scale set by AMSB and the point at which the FI term is zero, this choice requires an essentially arbitrary value for the FI term at the scale where the $U(1)'$ is broken. As suggested above, a more natural assumption is that the value would be zero while the $U(1)'$ remains unbroken and is only generated below this scale. Also, if one wished to consider the possibility that $\mathcal{G}_{SM}$ arises from the breaking of a higher symmetry (such as in §6), then the $U(1)_Y$ would not have a Fayet-Iliopoulos term above the scale at which this symmetry breaks since such a symmetry is necessarily non-abelian. A sensible choice for the high scale at which the FI term for the SM $U(1)$ is zero is one that corresponds to the scale at which the proposed Grand Unified Theories also break, which is generally $\mathcal{O}(10^{16})$GeV.

The remainder of this chapter, which relates closely to the first half of [35], shall discuss the implications of the dynamically generated FI term within the framework of the $U(1)'$ augmented AMSB and how the spectrum calculations are affected. Revised versions of these spectra will then be presented for the case where the $U(1)$ of $\mathcal{G}_{\rm SM}$ is zero at the GUT scale $M_{\rm GUT}$.

5.1 AMSB and the Renormalisation of ξ

The renormalisation of the Fayet-Iliopoulos term has been described in [38], [39] and [40]. In general

$$\beta_\xi = \frac{\beta_g}{g}\xi + \hat{\beta}_\xi, \tag{5.8}$$

where $\hat{\beta}_\xi$ is determined by tadpoles involving the D-field and is independent of ξ. The one and two-loop contributions to $\hat{\beta}_\xi$ are

$$\begin{aligned} 16\pi^2\hat{\beta}_\xi^{(1)} &= 2g_1 \mathrm{Tr}\left[\mathcal{Y}m^2\right], \qquad (5.9)\\ 16\pi^2\hat{\beta}_\xi^{(2)} &= -4g_1 \mathrm{Tr}\left[\mathcal{Y}m^2\gamma^{(1)}\right], \qquad (5.10)\end{aligned}$$

with the longer expression for the three-loop contribution given in the MSSM case in [40].

Consider the generalisation of (4.14) to include the possibility of two Fayet-Iliopoulos terms,

$$(\bar{m}^2)^i{}_j = \frac{1}{2}m_{3/2}^2 Q\frac{d}{dQ}\gamma^i{}_j + \xi(\mathcal{Y})^i{}_j + \xi'(\mathcal{Y}')^i{}_j, \tag{5.11}$$

where ξ and $\mathcal{Y}$ will be the FI term and hypercharge matrix of the $U(1)$ factor in $\mathcal{G}_{\rm SM}$ and ξ' and $\mathcal{Y}'$ are the equivalent terms for the $U(1)'$ described in previous chapters.

If ξ is assumed to be zero at gauge unification then the $U(1)'$ can be treated as in previous chapters at this scale (i.e. ξ' can be set equal to one without loss of generality and the charges of the MSSM fields are as in table 4.1.1). The evolution of this in running to lower energies is now determined by β_ξ which generates a non-zero ξ.

Equivalently, one might wish to retain the form of (4.14) rather than introducing a second FI term. This can be done by considering the evolution in the D-eliminated theory but any contributions to measurable physics will of course be the same in either approach.

In the previous treatment described in §4 it was obvious that the $U(1)'$ charges should be chosen so that L and e were both positive (with the FI term chosen to be positive) in order for both the lepton doublet and the lepton singlet to receive a positive contribution to their (mass)^2 and eliminate the tachyons. This is not necessarily still the case since the value of any scalar (mass)^2 in the low energy theory will depend on the contribution from *both* ξ and ξ' at that scale. Indeed the region which L and e may be chosen from at the unification scale in order to produce an acceptable low-energy spectrum does include some $L < 0$.

It should be noted that because of (5.3-5.7) it would be possible to obtain any spectrum using the treatment from §4 by choosing a *different* L and e provided that both calculations were performed to a single scale (rather than with masses calculated at the pole).

5.2 Calculation of Precision Sparticle Spectra

This follows much of the same procedure as in §4 except that without the assumption of RG-invariance one is forced to calculate values of the running coupling constants at the unification scale then run down from here to M_{SUSY}.

Figure (5.1) shows the region of (e, L) which produces an acceptable spectrum. This region is defined by the same experimental conditions as figure (4.3), with the mass of the CP-odd Higgs again responsible for the line to the upper-right of the plot.

5.3 Sparticle Spectra

The tables show spectra for several different choices of the input parameters. In each case, as before, the LSP is marked by an asterisk by its three-loop mass.

Since these spectra are likely to resemble real physics that might be observed in collider experiments (rather than those in §4 which were presented as the first evidence that realistic physics was possible with using this mechanism), it is worth considering how such spectra may be identified.

In all of these spectra the lightest neutralino and the lightest chargino have very similar masses - in each case the chargino is heavier by $\mathcal{O}(100)$MeV. This is one of the defining features of AMSB and is because (4.1) leads to $M_2 < M_1$; the lightest neutralino and chargino are mostly Wino. This will be a useful feature in distinguishing AMSB from other SUSY breaking scenarios in collider experiments.

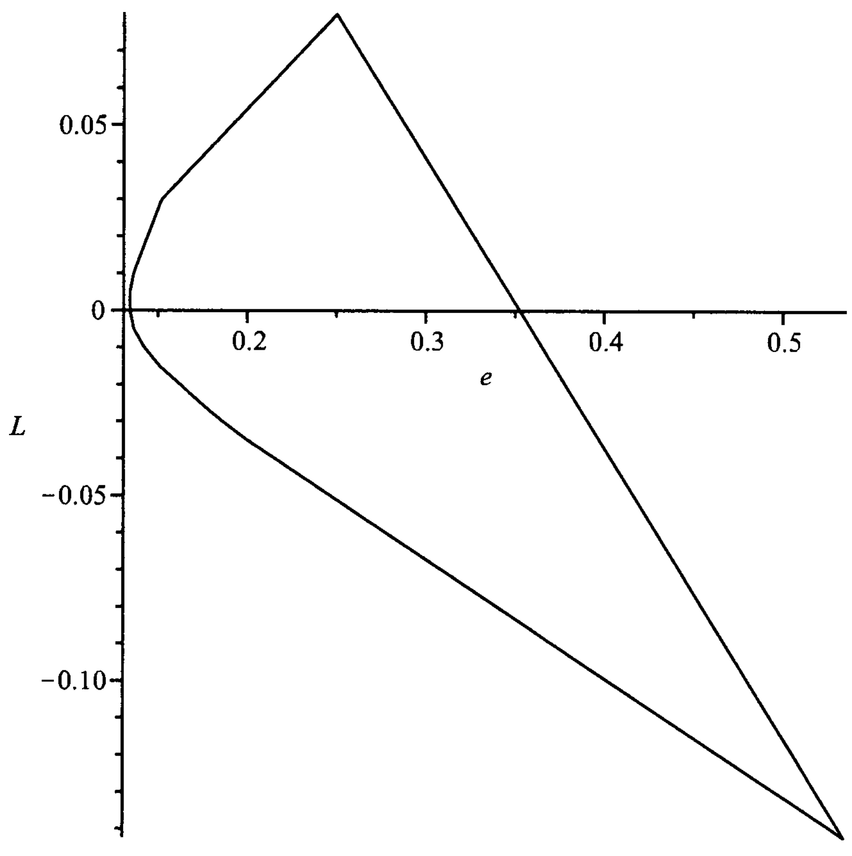

Figure 5.1: Allowed region of (e, L) for $m_{3/2} = 40\text{TeV}$, $\tan\beta = 10$ and $\mu > 0$

5.4 Mass Sum Rules

The following relations between masses will be a useful feature by which this scenario may be identified.

$$
\begin{aligned}
m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2 + m_{\tilde{b}_1}^2 + m_{\tilde{b}_2}^2 - 2m_t^2 &\approx 2.62 m_{\tilde{g}}^2, &(5.12)\\
m_{\tilde{\tau}_1}^2 + m_{\tilde{\tau}_2}^2 + m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2 - 2m_t^2 &\approx 1.05 m_{\tilde{g}}^2, &(5.13)\\
m_{\tilde{e}_L}^2 + 2m_{\tilde{u}_L}^2 + m_{\tilde{d}_L}^2 &\approx 2.53 m_{\tilde{g}}^2, &(5.14)\\
m_{\tilde{u}_R}^2 + m_{\tilde{d}_R}^2 + m_{\tilde{u}_L}^2 + m_{\tilde{d}_L}^2 &\approx 3.45 m_{\tilde{g}}^2, &(5.15)\\
m_{\tilde{u}_L}^2 + m_{\tilde{d}_L}^2 - m_{\tilde{u}_R}^2 - m_{\tilde{e}_R}^2 &\approx 0.87 m_{\tilde{g}}^2. &(5.16)
\end{aligned}
$$

These hold because the (e, L) contributions on the left-hand side cancel at tree level. The numerical coefficients on the right-hand side have been taken from table (5.3) and vary slightly with $\tan\beta$.

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	924	900	897
$\tilde{t}_1$	489	487	474
$\tilde{t}_2$	763	753	743
$\tilde{b}_1$	943	933	923
$\tilde{b}_2$	718	707	696
$\tilde{\tau}_1$	277	269	269
$\tilde{\tau}_2$	208	202	203
$\tilde{u}_R$	766	758	746
$\tilde{u}_L$	838	822	811
$\tilde{d}_R$	966	956	947
$\tilde{d}_L$	841	826	815
χ_1^0	107	131	132*
χ_2^0	355	362	362
χ_3^0	561	584	577
χ_4^0	571	594	587
$\chi_1^\pm$	107	132	132
$\chi_2^\pm$	569	592	585
h	115	115	115
H	239	290	275
A	239	290	275
$H^\pm$	253	301	287
$\tilde{e}_R$	271	259	259
$\tilde{e}_L$	229	229	229
$\tilde{\nu}_\tau$	210	210	210
$\tilde{\nu}_e$	214	214	214

Table 5.1: Spectrum for $m_{3/2} = 40\text{TeV}$, $L = -0.03$, $e = 0.3$, $\tan\beta = 15$ and $\mu < 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	925	899	897
$\tilde{t}_1$	469	469	455
$\tilde{t}_2$	757	747	737
$\tilde{b}_1$	981	971	962
$\tilde{b}_2$	715	703	692
$\tilde{\tau}_1$	315	315	315
$\tilde{\tau}_2$	241	226	226
$\tilde{u}_R$	754	746	733
$\tilde{u}_L$	828	813	801
$\tilde{d}_R$	991	981	972
$\tilde{d}_L$	832	816	805
χ_1^0	106	131	131*
χ_2^0	354	362	361
χ_3^0	533	558	550
χ_4^0	545	570	563
$\chi_1^\pm$	106	131	131
$\chi_2^\pm$	541	567	559
h	109	114	113
H	151	239	220
A	147	238	219
$H^\pm$	172	252	233
$\tilde{c}_R$	248	234	234
$\tilde{e}_L$	314	314	314
$\tilde{\nu}_\tau$	302	302	302
$\tilde{\nu}_e$	303	303	304

Table 5.2: Spectrum for $m_{3/2} = 40\text{TeV}$, $L = 0.01$, $e = 0.3$, $\tan\beta = 10$ and $\mu < 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	925	900	897
$\tilde{t}_1$	502	500	487
$\tilde{t}_2$	766	757	746
$\tilde{b}_1$	956	946	936
$\tilde{b}_2$	724	712	702
$\tilde{\tau}_1$	267	266	266
$\tilde{\tau}_2$	212	199	199
$\tilde{u}_R$	774	766	753
$\tilde{u}_L$	834	819	808
$\tilde{d}_R$	965	955	946
$\tilde{d}_L$	838	823	812
χ_1^0	106	131	131*
χ_2^0	354	362	362
χ_3^0	569	593	585
χ_4^0	580	604	596
$\chi_1^\pm$	107	131	131
$\chi_2^\pm$	577	601	594
h	114	114	114
H	333	373	361
A	333	373	361
$H^\pm$	342	381	370
$\tilde{e}_R$	225	212	212
$\tilde{e}_L$	262	261	262
$\tilde{\nu}_\tau$	248	247	247
$\tilde{\nu}_e$	250	249	249

Table 5.3: Spectrum for $m_{3/2} = 40$TeV, $L = 0$, $c = 1/4$, $\tan\beta = 10$ and $\mu > 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	925	900	897
$\tilde{t}_1$	475	475	461
$\tilde{t}_2$	768	757	747
$\tilde{b}_1$	963	953	943
$\tilde{b}_2$	727	716	705
$\tilde{\tau}_1$	305	294	294
$\tilde{\tau}_2$	211	211	211
$\tilde{u}_R$	755	747	734
$\tilde{u}_L$	839	823	812
$\tilde{d}_R$	973	963	953
$\tilde{d}_L$	842	827	816
χ_1^0	106	131	131*
χ_2^0	354	362	362
χ_3^0	550	575	567
χ_4^0	562	586	579
$\chi_1^\pm$	106	131	131
$\chi_2^\pm$	558	583	576
h	114	114	114
H	256	311	297
A	256	311	296
$H^\pm$	267	321	307
$\tilde{e}_R$	306	294	294
$\tilde{e}_L$	217	218	219
$\tilde{\nu}_\tau$	200	201	201
$\tilde{\nu}_e$	202	203	203

Table 5.4: Spectrum for $m_{3/2} = 40$TeV, $L = -0.05$, $e = 0.35$, $\tan\beta = 10$ and $\mu > 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	925	899	897
$\tilde{t}_1$	486	485	471
$\tilde{t}_2$	758	747	737
$\tilde{b}_1$	974	964	955
$\tilde{b}_2$	714	703	692
$\tilde{\tau}_1$	322	321	321
$\tilde{\tau}_2$	196	180	180
$\tilde{u}_R$	765	756	744
$\tilde{u}_L$	827	812	800
$\tilde{d}_R$	984	974	965
$\tilde{d}_L$	831	815	804
χ_1^0	106	131	131*
χ_2^0	354	362	362
χ_3^0	547	571	563
χ_4^0	558	583	575
$\chi_1^\pm$	106	131	131
$\chi_2^\pm$	555	579	572
h	114	114	114
H	239	295	280
A	238	295	280
$H^\pm$	252	306	291
$\tilde{e}_R$	204	189	188
$\tilde{e}_L$	322	321	321
$\tilde{\nu}_\tau$	310	309	309
$\tilde{\nu}_e$	311	311	311

Table 5.5: Spectrum for $m_{3/2} = 40$TeV, $L = 0.03$, $e = 0.25$, $\tan\beta = 10$ and $\mu > 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	924	899	897
$\tilde{t}_1$	499	497	484
$\tilde{t}_2$	761	751	741
$\tilde{b}_1$	943	932	923
$\tilde{b}_2$	715	704	693
$\tilde{\tau}_1$	212	270	270
$\tilde{\tau}_2$	198	185	185
$\tilde{u}_R$	774	766	753
$\tilde{u}_L$	834	819	808
$\tilde{d}_R$	965	955	946
$\tilde{d}_L$	838	823	812
χ_1^0	107	132	132*
χ_2^0	355	362	362
χ_3^0	567	590	582
χ_4^0	576	599	592
$\chi_1^\pm$	107	132	132
$\chi_2^\pm$	574	597	590
h	115	115	115
H	266	311	297
A	266	311	297
$H^\pm$	279	321	308
$\tilde{e}_R$	225	212	212
$\tilde{e}_L$	263	262	262
$\tilde{\nu}_\tau$	246	245	245
$\tilde{\nu}_e$	250	249	249

Table 5.6: Spectrum for $m_{3/2} = 40\text{TeV}$, $L = 0$, $c = 1/4$, $\tan\beta = 15$ and $\mu > 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	924	900	897
$\tilde{t}_1$	489	487	474
$\tilde{t}_2$	763	753	743
$\tilde{b}_1$	943	933	923
$\tilde{b}_2$	718	707	696
$\tilde{\tau}_1$	277	269	269
$\tilde{\tau}_2$	208	202	203
$\tilde{u}_R$	766	758	746
$\tilde{u}_L$	838	822	811
$\tilde{d}_R$	966	956	947
$\tilde{d}_L$	841	826	815
χ_1^0	107	131	132*
χ_2^0	355	362	362
χ_3^0	561	584	577
χ_4^0	571	594	587
$\chi_1^\pm$	107	132	132
$\chi_2^\pm$	569	592	585
h	115	115	115
H	239	290	275
A	239	290	275
$H^\pm$	253	301	287
$\tilde{e}_R$	271	259	259
$\tilde{e}_L$	229	229	229
$\tilde{\nu}_\tau$	210	209	210
$\tilde{\nu}_e$	214	214	214

Table 5.7: Spectrum for $m_{3/2} = 40\text{TeV}$, $L = -0.03$, $e = 0.3$, $\tan\beta = 15$ and $\mu > 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	924	899	897
$\tilde{t}_1$	504	502	489
$\tilde{t}_2$	759	750	739
$\tilde{b}_1$	944	934	924
$\tilde{b}_2$	712	701	690
$\tilde{\tau}_1$	288	287	287
$\tilde{\tau}_2$	168	151	151
$\tilde{u}_R$	778	770	758
$\tilde{u}_L$	832	816	805
$\tilde{d}_R$	966	957	947
$\tilde{d}_L$	836	820	809
χ_1^0	107	132	132*
χ_2^0	355	363	363
χ_3^0	569	591	584
χ_4^0	578	601	594
$\chi_1^\pm$	107	132	132
$\chi_2^\pm$	576	599	592
h	115	115	115
H	275	317	303
A	275	317	314
$H^\pm$	287	327	314
$\tilde{e}_R$	190	175	175
$\tilde{e}_L$	285	284	284
$\tilde{\nu}_\tau$	269	268	268
$\tilde{\nu}_e$	273	272	272

Table 5.8: Spectrum for $m_{3/2} = 40$TeV, $L = 0.02$, $e = 0.22$, $\tan\beta = 15$ and $\mu > 0$

Chapter 6

AMSB in MSSM$\otimes U(1)'$ and Grand Unified Theories

One of the most often mentioned motivations for supersymmetry at low energies is the way in which the three separate coupling constants of $\mathcal{G}_{\mathrm{SM}}$ can be made to converge at a common scale, M_{GUT}, typically $\mathcal{O}(10^{16})$GeV. This is attractive because it allows physicists to consider the possibility that $\mathcal{G}_{\mathrm{SM}}$ arises from the spontaneous breaking of a higher symmetry at M_{GUT}. Such theories bear the collective name *Grand Unified Theories*, or *GUTs*. For a review of GUTs see [26].

Note that while it is possible for GUTs to be constructed without supersymmetry at any scale, the field content of the Standard Model in insufficient. Supersymmetry provides a natural framework in which GUTs can be constructed without demanding additional fields that themselves have no motivation other than to allow the coupling constants to unify.

This has been mentioned in passing in §2 and the scale M_{GUT} was used as a natural cutoff in §5. It is reasonable now to consider how the model of $U(1)'$ augmented MSSM from AMSB may be compatible with a simple Grand Unified Theory. This idea has been explored in the second half of [35] using the example of a unified $SU(5)$ gauge group and will now be discussed here.

6.1 $SU(5)$ GUT

There are to be n_f sets of matter fields, with the Q, u^c and e forming a 10 and the L and d^c form a $\bar{5}$. There are then n_h sets of Higgs doublets (as the ones in the MSSM) and Higgs triplets whose MSSM charges are $(3,1)_{-2/3}$ and $(\bar{3},1)_{2/3}$. These form a 5 and a $\bar{5}$ so the triplets share the same $U(1)'$ charges as the Higgs doublets. For an $SU(5)\otimes U(1)'$ embedding the $U(1)'$ charges must then satisfy

$$Q = u^c = e, \tag{6.1}$$

$$d^c = L, \tag{6.2}$$

while $U(1)'$ invariance of the Yukawa terms demands

$$h_1 = -L - e, \tag{6.3}$$

$$h_2 = -2e, \tag{6.4}$$

$$\nu^c = 2e - L. \tag{6.5}$$

Using these charge assignments, one can begin to consider the $U(1)'^3$, $U(1)'$-gravity anomalies and the mixed anomalies involving the $U(1)'$ and $\mathcal{G}_{SM}$.

$$\begin{aligned}
A(SU(3)^2U(1)') &= \left(2\times 3^2 e + 3^2 e + 3^2 L\right) n_f \\
&+ \left(3^2(-e-L) + 3^2(-2e)\right) n_h, \\
&= 9\left((L+3e)n_f - (L+3e)n_h\right), && (6.6)\\
A(SU(2)^2U(1)') &= \left(3\times 2^2 e + 2^2 L\right) n_f \\
&+ \left(2^2(-e-L) + 2^2(-2e)\right) n_h, \\
&= 4\left((L+3e)n_f - (L+3e)n_h\right), && (6.7)\\
A(U(1)^2U(1)') &= \left(3\times 2\times\left(\frac{1}{3}\right)^2 e + 3\times\left(-\frac{4}{3}\right)^2 e\right. \\
&+ \left. 3\times\left(\frac{2}{3}\right)^2 L + 2\times(-1)^2 L + 2^2 e\right) n_f \\
&+ \left(2\times(-1)^2(-e-L) + 2\times 1^2(-2e)\right. \\
&+ \left. 3\times\left(\frac{2}{3}\right)^2(-e-L) + 3\times\left(-\frac{2}{3}\right)^2(-2e)\right) n_h, \\
&= \frac{10}{3}\left((L+3e)n_f - (L+3e)n_h\right), && (6.8)\\
A(U(1)'^2U(1)) &= \left(3\times 2\times\left(\frac{1}{3}\right)e^2 + 3\times\left(-\frac{4}{3}\right)e^2 + 3\times\left(\frac{2}{3}\right)L^2\right. \\
&+ \left. 2\times(-1)L^2 + 2e^2\right) n_f \\
&+ \left(2\times(-1)(-e-L) + 2\times 1(-2e)\right. \\
&+ \left. 3\times\left(\frac{2}{3}\right)(-e-L) + 3\times\left(-\frac{2}{3}\right)(-2e)\right) n_h, \\
&= 0, && (6.9)\\
A(U(1)'^3) &= \left(3\times 2e^3 + 3e^3 + 3L^3 + 2L^3 + e^3 + (2e-L)^3\right) n_f \\
&+ \left(2(-e-L)^3 + 2(-2e)^3 + 3(-e-L)^3 + 3(-2e)^3\right) n_h, \\
&= \left(10e^3 + 5L^3 + (2e-L)^3\right) n_f \\
&- \left(40e^3 + 5(e+L)^3\right) n_h, && (6.10)\\
A(U(1)'\text{-grav}) &= \left(3\times 2e + 3e + 3L + 2L + e + (2e-L)\right) n_f \\
&+ \left(2(-e-L) + 2(-2e) + 3(-e-L) + 3(-2e)\right) n_h, \\
&= 4(L+3e)n_f - 5(L+3e)n_h. && (6.11)
\end{aligned}$$

The $SU(3)^2U(1)'$, $SU(2)^2U(1)'$ and $U(1)^2U(1)'$ anomalies are proportional to

$$A_1 = (n_f - n_h)(L + 3e), \tag{6.12}$$

the $U(1)'^2U(1)$ anomaly is zero, the $U(1)'^3$ anomaly is equal to

$$A_3 = (L + 3e)[5(n_f - n_h)(L^2 + 3e^2) - n_f(L + 3e)^2] \tag{6.13}$$

and the $U(1)'$-gravity anomaly is proportional to

$$A_G = (L + 3e)(4n_f - 5n_h). \tag{6.14}$$

Notice that if $L = -3e$ this is anomaly free for arbitrary n_f and n_h. In this case the $U(1)'$ is the additional $U(1)$ factor when $SO(10)$ is broken to $SU(5)$ (i.e. $SO(10) \to SU(5) \otimes U(1)'$) [36]. In this case the matter fields form a 16 and the Higgses a 10 under $SO(10)$. Though the appeal of this case is obvious the line $L = -3e$ does not pass through the region of (e, L) space which renders the theory free from tachyons (see figure (6.1)).

If $n_f = n_h$ then A_1 vanishes, so that the theory can now be made anomaly free by the inclusion of fields which are singlets under $\mathcal{G}_{\mathrm{SM}}$. The contributions which must be cancelled are now

$$\begin{aligned} A_3 &= -n_f(L + 3e)^3, && (6.15) \\ A_G &= -n_f(L + 3e). && (6.16) \end{aligned}$$

This is dealt with very simply by the addition of n_f sets of fields N which have the common $U(1)'$ charge of $(L + 3e)$. These N can function as right handed neutrinos. The complete set of charge assignments is given in table (6.1).

10	$\bar{5}$	ν^c	H	$\bar{H}$	N
e	L	$2e - L$	$-2e$	$-e - L$	$L + 3e$

Table 6.1: Charges for anomaly-free $U(1)'$ with $SU(5)$ unification

For simplicity, consider now the low-energy scenario which is most similar to that which we have already discussed, i.e. that below the unification scale we have the usual MSSM effective field theory with three generations of matter fields and a single pair of Higgs doublets, and that the soft terms are determined by AMSB with a Fayet-Iliopoulos term associated with the $U(1)'$. This gives the usual AMSB relations with set of equations (4.14) given explicitly as

$$\bar{m}_Q^2 = m_Q^2 + e\xi, \tag{6.17}$$

$$\bar{m}_{tc}^2 = m_{tc}^2 + e\xi, \tag{6.18}$$
$$\bar{m}_{\tau c}^2 = m_{\tau c}^2 + e\xi, \tag{6.19}$$
$$\bar{m}_{bc}^2 = m_{bc}^2 + L\xi, \tag{6.20}$$
$$\bar{m}_{L}^2 = m_{L}^2 + L\xi, \tag{6.21}$$
$$\bar{m}_{H_1}^2 = m_{H_1}^2 - (e + L)\xi, \tag{6.22}$$
$$\bar{m}_{H_2}^2 = m_{H_2}^2 - 2e\xi. \tag{6.23}$$

6.2 Sparticle Spectra

Figure (6.1) shows the region of (e, L) which produces an acceptable spectrum. The plot is defined by the same experimental constraints as figures (4.3) and (5.1). Notice that in this case the line corresponding to the CP-odd Higgs mass reaching its experimental bound only crosses the one set by the slepton masses once; the region is unbounded on one side. Larger values of either e or L lead to a larger Higgs μ-term which means that a higher degree of fine-tuning is needed in order to obtain the correct electroweak ground state, for this reason the spectra presented here are for relatively small values of e and L.

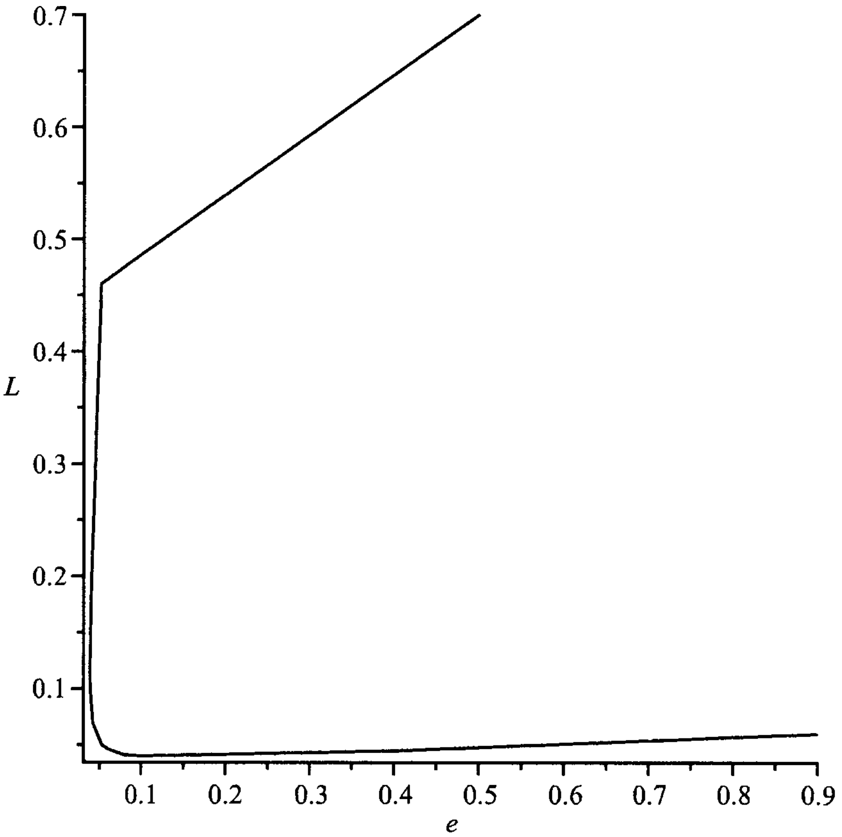

Figure 6.1: Allowed region of (e, L) for $m_{3/2} = 40\text{TeV}$, $\tan\beta = 15$ and $\mu > 0$

6.3 Mass Sum Rules

A set of relations between masses similar to (5.12-5.16) can also be found for this case. Again the (e, L) contributions to these cancel, but the numerical coefficients vary slightly with $\tan\beta$. The numerical coefficients given here were taken from table (6.2).

$$m^2_{\tilde{u}_L} + m^2_{\tilde{d}_L} - m^2_{\tilde{u}_R} - m^2_{\tilde{e}_R} \approx 0.85 m^2_{\tilde{g}}, \tag{6.24}$$

$$m^2_{\tilde{b}_1} + m^2_{\tilde{b}_2} - m^2_{\tilde{\tau}_1} - m^2_{\tilde{\tau}_2} \approx 1.44 m^2_{\tilde{g}}, \tag{6.25}$$

$$m^2_{\tilde{b}_1} + m^2_{\tilde{b}_2} - m^2_{\tilde{e}_L} - m^2_{\tilde{e}_R} \approx 1.43 m^2_{\tilde{g}}, \tag{6.26}$$

$$m^2_{\tilde{d}_L} + m^2_{\tilde{d}_R} - m^2_{\tilde{e}_L} - m^2_{\tilde{e}_L} - m^2_{\tilde{e}_R} \approx 1.68 m^2_{\tilde{g}}. \tag{6.27}$$

The tables show spectra for several different choices of the input parameters. In each case, as before, the LSP is marked by an asterisk by its three-loop mass.

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	945	919	917
$\tilde{t}_1$	760	750	742
$\tilde{t}_2$	914	901	894
$\tilde{b}_1$	947	934	924
$\tilde{b}_2$	861	846	838
$\tilde{\tau}_1$	447	444	444
$\tilde{\tau}_2$	391	387	388
$\tilde{u}_R$	960	941	932
$\tilde{u}_L$	962	949	939
$\tilde{d}_R$	967	954	945
$\tilde{d}_L$	963	945	936
χ^0_1	110	134	135*
χ^0_2	360	368	368
χ^0_3	931	942	937
χ^0_4	935	946	942
$\chi^\pm_1$	110	134	135
$\chi^\pm_2$	936	946	942
h	115	115	115
H	621	632	625
A	622	633	626
$H^\pm$	627	638	631
$\tilde{e}_R$	424	422	422
$\tilde{e}_L$	424	419	419
$\tilde{\nu}_\tau$	413	408	408
$\tilde{\nu}_e$	416	411	411

Table 6.2: Spectrum for $m_{3/2} = 40$TeV, $L = 1/5$, $e = 1/5$, $\tan\beta = 15$ and $\mu > 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	943	919	916
$\tilde{t}_1$	756	746	739
$\tilde{t}_2$	898	886	878
$\tilde{b}_1$	910	896	887
$\tilde{b}_2$	821	804	795
$\tilde{\tau}_1$	462	460	460
$\tilde{\tau}_2$	366	362	363
$\tilde{u}_R$	960	942	933
$\tilde{u}_L$	963	949	940
$\tilde{d}_R$	968	955	945
$\tilde{d}_L$	963	945	936
χ_1^0	110	134	134*
χ_2^0	360	367	367
χ_3^0	919	929	924
χ_4^0	923	933	928
$\chi_1^\pm$	110	134	134
$\chi_2^\pm$	923	933	928
h	115	115	115
H	473	475	466
A	473	476	466
$H^\pm$	481	483	474
$\tilde{e}_R$	423	422	422
$\tilde{e}_L$	424	419	419
$\tilde{\nu}_\tau$	411	406	406
$\tilde{\nu}_e$	416	411	411

Table 6.3: Spectrum for $m_{3/2} = 40$TeV, $L = 1/5$, $e = 1/5$, $\tan\beta = 25$ and $\mu > 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	966	941	939
$\tilde{t}_1$	983	969	964
$\tilde{t}_2$	1093	1078	1072
$\tilde{b}_1$	1060	1043	1037
$\tilde{b}_2$	869	846	835
$\tilde{\tau}_1$	772	772	772
$\tilde{\tau}_2$	399	391	391
$\tilde{u}_R$	1143	1126	1118
$\tilde{u}_L$	1148	1127	1119
$\tilde{d}_R$	1150	1130	1122
$\tilde{d}_L$	968	948	939
χ_1^0	111	135	135*
χ_2^0	362	369	369
χ_3^0	1269	1274	1271
χ_4^0	1270	1275	1272
$\chi_1^\pm$	111	135	135
$\chi_2^\pm$	1272	1277	1273
h	116	116	116
H	759	755	750
A	759	756	750
$H^\pm$	764	760	754
$\tilde{c}_R$	774	773	773
$\tilde{e}_L$	412	404	404
$\tilde{\nu}_\tau$	399	391	392
$\tilde{\nu}_e$	404	396	396

Table 6.4: Spectrum for $m_{3/2} = 40\text{TeV}$, $L = 1/5$, $c = 3/5$, $\tan\beta = 25$ and $\mu > 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	948	923	920
$\tilde{t}_1$	758	747	739
$\tilde{t}_2$	906	893	885
$\tilde{b}_1$	1025	1011	1002
$\tilde{b}_2$	849	832	824
$\tilde{\tau}_1$	621	618	618
$\tilde{\tau}_2$	399	396	396
$\tilde{u}_R$	958	939	930
$\tilde{u}_L$	965	951	942
$\tilde{d}_R$	1063	1050	1042
$\tilde{d}_L$	962	942	933
χ_1^0	110	135	135*
χ_2^0	361	368	368
χ_3^0	928	938	934
χ_4^0	933	943	938
$\chi_1^\pm$	111	135	135
$\chi_2^\pm$	933	943	938
h	115	115	115
H	357	371	359
A	357	371	359
$H^\pm$	366	380	369
$\tilde{e}_R$	412	410	410
$\tilde{e}_L$	621	617	617
$\tilde{\nu}_\tau$	612	609	609
$\tilde{\nu}_e$	615	611	612

Table 6.5: Spectrum for $m_{3/2} = 40$TeV, $L = 0.4$, $e = 0.2$, $\tan\beta = 20$ and $\mu > 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	965	939	936
$\tilde{t}_1$	989	975	970
$\tilde{t}_2$	1111	1096	1089
$\tilde{b}_1$	1079	1062	1055
$\tilde{b}_2$	904	883	873
$\tilde{\tau}_1$	776	775	775
$\tilde{\tau}_2$	256	243	243
$\tilde{u}_R$	1141	1124	1116
$\tilde{u}_L$	1148	1127	1120
$\tilde{d}_R$	1151	1130	1122
$\tilde{d}_L$	916	896	885
χ_1^0	110	134	135*
χ_2^0	361	369	369
χ_3^0	1278	1291	1288
χ_4^0	1288	1294	1290
$\chi_1^\pm$	110	135	135
$\chi_2^\pm$	1288	1294	1291
h	114	114	114
H	961	966	961
A	960	965	961
$H^\pm$	964	969	964
$\tilde{e}_R$	777	776	776
$\tilde{e}_L$	260	246	247
$\tilde{\nu}_\tau$	245	231	231
$\tilde{\nu}_e$	247	233	234

Table 6.6: Spectrum for $m_{3/2} = 40\text{TeV}$, $L = 0.1$, $e = 0.6$, $\tan\beta = 10$ and $\mu > 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	956	930	928
$\tilde{t}_1$	824	812	805
$\tilde{t}_2$	966	951	943
$\tilde{b}_1$	1066	1051	1042
$\tilde{b}_2$	917	899	891
$\tilde{\tau}_1$	660	656	656
$\tilde{\tau}_2$	514	511	512
$\tilde{u}_R$	1008	987	979
$\tilde{u}_L$	1013	998	989
$\tilde{d}_R$	1086	1071	1063
$\tilde{d}_L$	1011	991	982
χ_1^0	110	135	135*
χ_2^0	361	369	369
χ_3^0	1031	1040	1036
χ_4^0	1035	1043	1039
$\chi_1^\pm$	111	135	135
$\chi_2^\pm$	1035	1044	1040
h	115	115	115
H	490	503	494
A	490	503	495
$H^\pm$	497	510	501
$\tilde{e}_R$	522	520	520
$\tilde{e}_L$	659	655	655
$\tilde{\nu}_\tau$	652	648	648
$\tilde{\nu}_e$	654	650	650

Table 6.7: Spectrum for $m_{3/2} = 40\text{TeV}$, $L = 0.45$, $e = 0.3$, $\tan\beta = 15$ and $\mu > 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	945	920	917
$\tilde{t}_1$	764	754	747
$\tilde{t}_2$	917	905	897
$\tilde{b}_1$	959	945	936
$\tilde{b}_2$	871	857	848
$\tilde{\tau}_1$	440	437	437
$\tilde{\tau}_2$	404	400	400
$\tilde{u}_R$	960	941	932
$\tilde{u}_L$	962	949	939
$\tilde{d}_R$	967	954	944
$\tilde{d}_L$	963	945	935
χ_1^0	109	134	134*
χ_2^0	360	367	367
χ_3^0	934	945	941
χ_4^0	939	950	946
$\chi_1^\pm$	109	134	134
$\chi_2^\pm$	939	950	946
h	114	114	114
H	660	672	666
A	660	672	666
$H^\pm$	665	677	671
$\tilde{e}_R$	424	422	422
$\tilde{e}_L$	424	419	419
$\tilde{\nu}_\tau$	415	410	410
$\tilde{\nu}_e$	416	411	411

Table 6.8: Spectrum for $m_{3/2} = 40$TeV, $L = 0.2$, $e = 0.2$, $\tan\beta = 10$ and $\mu > 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	951	926	923
$\tilde{t}_1$	823	812	806
$\tilde{t}_2$	967	954	946
$\tilde{b}_1$	955	939	930
$\tilde{b}_2$	910	893	885
$\tilde{\tau}_1$	534	533	533
$\tilde{\tau}_2$	411	405	405
$\tilde{u}_R$	1010	990	982
$\tilde{u}_L$	1010	996	987
$\tilde{d}_R$	1013	994	985
$\tilde{d}_L$	967	952	943
χ_1^0	110	135	135*
χ_2^0	361	368	368
χ_3^0	1030	1039	1035
χ_4^0	1033	1042	1038
$\chi_1^\pm$	110	135	135
$\chi_2^\pm$	1033	1042	1038
h	115	115	115
H	693	702	695
A	693	702	696
$H^\pm$	698	707	700
$\tilde{e}_R$	533	532	532
$\tilde{e}_L$	421	415	415
$\tilde{\nu}_\tau$	411	404	405
$\tilde{\nu}_e$	413	407	407

Table 6.9: Spectrum for $m_{3/2} = 40\text{TeV}$, $L = 0.2$, $e = 0.3$, $\tan\beta = 15$ and $\mu < 0$

Chapter 7

Tachyon Free AMSB Without $U(1)'$

An alternative mechanism for producing a realistic low energy spectrum with anomaly mediation as the dominant source of supersymmetry breaking was proposed in [41]. This approach is motivated by a string theory scenario in which the MSSM is located on a stack of branes while interaction between the brane on which $U(1)_Y$ is located and a hidden sector brane leads to the Bino receiving a large, non-AMSB contribution to its mass parameter. Most of the AMSB relations are retained but (4.1) is replaced by

$$M_1 = m_{3/2}\frac{\beta_{g_1}}{g_1} + \alpha_1 m_{3/2}, \tag{7.1}$$

$$M_a = m_{3/2}\frac{\beta_{g_a}}{g_a}, \qquad a = 2, 3. \tag{7.2}$$

For a suitable choice for α_1 this can lead to a tachyon-free, TeV-scale sparticle spectrum without any need for a Fayet-Iliopoulos term, since the large M_1 can give significant contributions to the scalar masses through the renormalisation group.

This is of course interesting from a phenomenological point of view, and one might readily imagine that other high-energy motivations exist whereby some other combination of the more general form

$$M_i = m_{3/2}\frac{\beta_{g_i}}{g_i} + \alpha_i m_{3/2}, \qquad i = 1, 2, 3 \tag{7.3}$$

could be obtained. How such models might arise shall not be considered here though they are at least as deserving of attention as the case where a common term was added to all scalar (mass)^2 that has been the main method for studying AMSB spectra in the past.

In this form this is highly unpredictive, but there are a number of realistic scenarios which might be considered within this huge set of possibilities. Aside from being interesting in their own right, it is important to consider how such models might be distinguished from those described in previous chapters.

It is easy to imagine models where any number of the α_i are zero while others are equal to each other. Perhaps the cases most deserving of study are those where all three of the α_i are equal to each other, and those where only one of them is non-zero, since it seems likely that there might exist some high energy mechanism which leads to these situations. If all the $\alpha_i = 0$ then the model is pure AMSB - including its tachyons, while $\alpha_2 = \alpha_3 = 0, \alpha_1 \neq 0$ corresponds to the model described in [41].

Consideration of the other cases where only one of the α_i is non-zero leads quickly to the conclusion that a realistic spectrum cannot possibly be obtained without some other source of SUSY breaking.

For $\alpha_3 \neq 0$ this is obvious since the problem of tachyons is only associated with those sfermions that are singlets under $SU(3)$ anyway (the asymptotic freedom of the strong coupling constant rescues the squarks as explained in §4). The renormalisation group will only communicate the effects of a large M_3 to the tachyonic slepton sector at higher orders, and this is insufficient to cure the problem.

For $\alpha_2 \neq 0$ the slepton doublet does - for suitable choice of α_2 - obtain a positive $(\text{mass})^2$ but for the singlet there is the same problem as for $\alpha_3 \neq 0$ since the right-handed slepton singlet couples only to the $U(1)$.

This leaves two appealing pictures which can produce realistic spectra. These are the 'Hypercharged Anomaly Mediation' suggested in [41] with $\alpha_1 \neq 0, \alpha_2 = \alpha_3 = 0$ and the case $\alpha_1 = \alpha_2 = \alpha_3 \neq 0$. Each of these cases works for a different region of $(\alpha_1, m_{3/2})$ and has slightly different phenomenology. Spectra for these models shall now be presented along with a brief discussion of how these could be distinguished from the Fayet-Iliopoulos type models of previous chapters.

7.1 Sparticle Spectra for Hypercharged Anomaly Mediation

In this section $\alpha_1 \neq 0, \alpha_2 = \alpha_3 = 0$

These spectra are similar to those presented in the FI chapters because they are still determined by AMSB. They still have a light Wino triplet, however they differ from the other scenarios enough that it will be possible to distinguish this mechanism from the Fayet-Iliopoulos one if enough experimental data is gathered.

Perhaps the most obvious feature is the heavy χ_4^0 which is mostly Bino. The reason for this is simple since a large term was added to the Bino mass parameter. Also, since the right-handed and left-handed fields couple differently to $U(1)_Y$ their masses are affected differently by this mechanism. This leads to much larger differences in mass between the right-handed and left-handed particles than is seen in the other scenarios.

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	945	923	920
$\tilde{t}_1$	643	626	613
$\tilde{t}_2$	1096	1074	1069
$\tilde{b}_1$	886	864	854
$\tilde{b}_2$	636	617	604
$\tilde{\tau}_1$	1454	1424	1425
$\tilde{\tau}_2$	691	670	670
$\tilde{u}_R$	886	866	856
$\tilde{u}_L$	1316	1287	1280
$\tilde{d}_R$	994	977	968
$\tilde{d}_L$	889	869	859
χ_1^0	108	131	131*
χ_2^0	522	553	546
χ_3^0	525	556	549
χ_4^0	1745	1699	1699
$\chi_1^{\pm}$	108	131	131
$\chi_2^{\pm}$	531	561	555
h	115	115	115
H	659	642	638
A	661	644	640
$H^{\pm}$	664	649	644
$\tilde{e}_R$	1501	1471	1472
$\tilde{e}_L$	740	719	719
$\tilde{\nu}_\tau$	687	665	665
$\tilde{\nu}_e$	735	714	715

Table 7.1: Spectrum for $m_{3/2} = 40\text{TeV}$, $\tan\beta = 30$, $\mu > 0$, $\alpha_1 = \frac{1}{4\pi}$, $\alpha_2 = \alpha_3 = 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	944	923	921
$\tilde{t}_1$	599	587	576
$\tilde{t}_2$	1102	1082	1077
$\tilde{b}_1$	811	796	787
$\tilde{b}_2$	582	567	554
$\tilde{\tau}_1$	1407	1376	1376
$\tilde{\tau}_2$	642	618	618
$\tilde{u}_R$	886	867	857
$\tilde{u}_L$	1317	1288	1281
$\tilde{d}_R$	994	978	969
$\tilde{d}_L$	890	871	861
χ_1^0	108	129	129*
χ_2^0	494	521	514
χ_3^0	497	525	518
χ_4^0	1744	1701	1701
$\chi_1^\pm$	108	129	129
$\chi_2^\pm$	503	530	523
h	115	114	114
H	387	352	349
A	389	354	351
$H^\pm$	397	363	360
$\tilde{c}_R$	1502	1471	1472
$\tilde{e}_L$	740	719	720
$\tilde{\nu}_\tau$	638	613	613
$\tilde{\nu}_e$	736	714	715

Table 7.2: Spectrum for $m_{3/2} = 40\text{TeV}$, $\tan\beta = 40$, $\mu > 0$, $\alpha_1 = \frac{1}{4\pi}$, $\alpha_2 = \alpha_3 = 0$

7.2 Sparticle Spectra with Heavy Gauginos

In this section $\alpha_1 = \alpha_2 = \alpha_3 \neq 0$.

When $m_{3/2} \approx 40$ as is typically the case in AMSB models, this produces an acceptable light Higgs mass, but very heavy sparticles. In table (7.5) the LSP has a mass of 443GeV compared with around 130GeV in most of the other situations. The gap between the masses of the lightest neutralino and chargino is not as small because M_2

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	946	923	920
$\tilde{t}_1$	683	667	655
$\tilde{t}_2$	1092	1070	1065
$\tilde{b}_1$	949	930	921
$\tilde{b}_2$	680	664	652
$\tilde{\tau}_1$	1481	1451	1452
$\tilde{\tau}_2$	719	698	699
$\tilde{u}_R$	885	864	855
$\tilde{u}_L$	1316	1286	1280
$\tilde{d}_R$	994	976	967
$\tilde{d}_L$	889	869	858
χ_1^0	108	131	131*
χ_2^0	538	570	564
χ_3^0	541	573	567
χ_4^0	1746	1699	1699
$\chi_1^\pm$	108	131	132
$\chi_2^\pm$	547	578	572
h	116	115	115
H	817	814	810
A	818	815	812
$H^\pm$	821	819	815
$\tilde{e}_R$	1501	1471	1472
$\tilde{e}_L$	740	719	719
$\tilde{\nu}_\tau$	715	693	694
$\tilde{\nu}_e$	735	714	714

Table 7.3: Spectrum for $m_{3/2} = 40$TeV, $\tan\beta = 20$, $\mu > 0$, $\alpha_1 = \frac{1}{4\pi}$, $\alpha_2 = \alpha_3 = 0$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	947	924	921
$\tilde{t}_1$	705	690	679
$\tilde{t}_2$	1090	1067	1062
$\tilde{b}_1$	983	965	956
$\tilde{b}_2$	702	688	676
$\tilde{\tau}_1$	1496	1466	1467
$\tilde{\tau}_2$	735	714	714
$\tilde{u}_R$	885	865	854
$\tilde{u}_L$	1315	1286	1279
$\tilde{d}_R$	993	976	966
$\tilde{d}_L$	889	868	858
χ_1^0	106	130	130*
χ_2^0	554	586	580
χ_3^0	558	590	584
χ_4^0	1746	1699	1699
$\chi_1^\pm$	107	130	131
$\chi_2^\pm$	563	594	588
h	114	114	114
H	902	904	900
A	902	904	901
$H^\pm$	905	907	904
$\tilde{e}_R$	1501	1471	1472
$\tilde{e}_L$	740	719	719
$\tilde{\nu}_\tau$	730	709	709
$\tilde{\nu}_e$	735	714	714

Table 7.4: Spectrum for $m_{3/2} = 40\text{TeV}$, $\tan\beta = 10$, $\mu > 0$, $\alpha_1 = \frac{1}{4\pi}$, $\alpha_2 = \alpha_3 = 0$

received the same large contribution as M_1, and there is not the same splitting between the sfermion masses as in the Hypercharged AMSB case.

Notice that the tables from (7.7) have been produced with a lower $m_{3/2}$ than was used in typical AMSB models. This is perfectly acceptable since the higher value is only favoured in other cases because it yields $M_{\rm SUSY} \sim 1$TeV. This is interesting because it gives a lighter general spectrum with a light Higgs in the accepted range. As should be expected the neutralinos/charginos and the gluino are all fairly heavy compared with the models which used the Fayet-Iliopoulos mechanism, though not by as much as the heaviest neutralino in Hypercharged AMSB since α_1 is smaller here than it was there.

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	1040	1020	1016
$\tilde{t}_1$	755	749	738
$\tilde{t}_2$	1034	1021	1012
$\tilde{b}_1$	1011	998	989
$\tilde{b}_2$	945	939	927
$\tilde{\tau}_1$	646	641	640
$\tilde{\tau}_2$	507	499	499
$\tilde{u}_R$	993	985	974
$\tilde{u}_L$	1109	1094	1084
$\tilde{d}_R$	1112	1097	1086
$\tilde{d}_L$	951	945	933
χ_1^0	418	448	443*
χ_2^0	436	468	463
χ_3^0	717	702	702
χ_4^0	795	768	767
$\chi_1^\pm$	425	455	450
$\chi_2^\pm$	795	767	766
h	115	114	114
H	768	780	776
A	768	780	776
$H^\pm$	772	785	780
$\tilde{e}_R$	512	504	505
$\tilde{e}_L$	648	643	641
$\tilde{\nu}_\tau$	640	635	633
$\tilde{\nu}_e$	642	637	636

Table 7.5: Spectrum for $m_{3/2} = 40$TeV, $\tan\beta = 10$, $\mu > 0$, $\alpha_1 = \alpha_2 = \alpha_3 = \frac{1}{16\pi}$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	1039	1019	1016
$\tilde{t}_1$	757	752	741
$\tilde{t}_2$	1027	1015	1005
$\tilde{b}_1$	1006	993	983
$\tilde{b}_2$	928	922	910
$\tilde{\tau}_1$	641	635	634
$\tilde{\tau}_2$	490	481	482
$\tilde{u}_R$	994	985	974
$\tilde{u}_L$	1109	1094	1084
$\tilde{d}_R$	1112	1097	1087
$\tilde{d}_L$	951	945	933
χ_1^0	409	493	434*
χ_2^0	425	458	453
χ_3^0	717	702	701
χ_4^0	794	767	765
$\chi_1^\pm$	415	446	441
$\chi_2^\pm$	794	766	765
h	115	115	115
H	723	735	731
A	723	736	732
$H^\pm$	728	740	736
$\tilde{e}_R$	512	504	505
$\tilde{e}_L$	648	643	641
$\tilde{\nu}_\tau$	634	629	627
$\tilde{\nu}_e$	643	637	635

Table 7.6: Spectrum for $m_{3/2} = 40$TeV, $\tan\beta = 20$, $\mu > 0$, $\alpha_1 = \alpha_2 = \alpha_3 = \frac{1}{16\pi}$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	926	907	904
$\tilde{t}_1$	662	659	648
$\tilde{t}_2$	896	885	876
$\tilde{b}_1$	867	856	847
$\tilde{b}_2$	823	818	807
$\tilde{\tau}_1$	513	508	507
$\tilde{\tau}_2$	394	387	387
$\tilde{u}_R$	857	850	840
$\tilde{u}_L$	945	932	922
$\tilde{d}_R$	948	935	925
$\tilde{d}_L$	828	823	812
χ_1^0	364	388	384*
χ_2^0	391	420	415
χ_3^0	538	526	526
χ_4^0	634	611	611
$\chi_1^\pm$	374	399	395
$\chi_2^\pm$	633	611	610
h	114	114	113
H	633	646	642
A	633	646	642
$H^\pm$	639	651	647
$\tilde{e}_R$	398	392	392
$\tilde{e}_L$	514	509	508
$\tilde{\nu}_\tau$	506	501	500
$\tilde{\nu}_e$	508	503	501

Table 7.7: Spectrum for $m_{3/2} = 28\text{TeV}$, $\tan\beta = 10$, $\mu > 0$, $\alpha_1 = \alpha_2 = \alpha_3 = \frac{1}{14\pi}$

Mass/GeV	1-loop	2-loop	3-loop
$\tilde{g}$	895	877	834
$\tilde{t}_1$	640	636	626
$\tilde{t}_2$	868	858	849
$\tilde{b}_1$	838	827	818
$\tilde{b}_2$	795	791	780
$\tilde{\tau}_1$	496	491	489
$\tilde{\tau}_2$	380	373	374
$\tilde{u}_R$	829	822	812
$\tilde{u}_L$	913	900	891
$\tilde{d}_R$	916	904	894
$\tilde{d}_L$	800	796	785
χ_1^0	351	374	370*
χ_2^0	378	406	402
χ_3^0	519	507	507
χ_4^0	612	591	590
$\chi_1^\pm$	361	385	381
$\chi_2^\pm$	612	590	589
h	113	113	113
H	612	624	620
A	612	624	620
$H^\pm$	617	629	625
$\tilde{e}_R$	384	378	378
$\tilde{e}_L$	497	492	490
$\tilde{\nu}_\tau$	488	483	482
$\tilde{\nu}_e$	490	485	483

Table 7.8: Spectrum for $m_{3/2} = 27\text{TeV}$, $\tan\beta = 10$, $\mu > 0$, $\alpha_1 = \alpha_2 = \alpha_3 = \frac{1}{14\pi}$

Chapter 8

Conclusions

The Minimal Supersymmetric Standard Model is a promising framework for physics beyond the Standard Model. It has the potential to solve many of the problems present in the SM including the hierarchy problem and the convergence of the coupling constants to the point of potential Grand Unification. It is also of interest to cosmologists because it provides a candidate for the Dark Matter. It is important to understand the mechanism of supersymmetry breaking in order to make predictions within the MSSM.

Anomaly Mediated Supersymmetry Breaking describes the whole MSSM in sets of renormalisation group invariant equations that can be written on four short lines. This allows precise calculations to be made relatively simply. However, in its minimal form AMSB does not predict the observed electroweak ground state.

The correct electroweak ground state can be obtained from AMSB if the gauge group $\mathcal{G}_{SM}$ is extended by an abelian factor $U(1)'$ which has a large Fayet-Iliopoulos term and the $U(1)'$ is broken at the high scale. The high scale physics decouples from the low energy theory in a very elegant manner leaving only the MSSM.

The mechanism involving the $U(1)'$ was originally believed to preserve the renormalisation group invariance of AMSB. However this would only be the case if $\mathcal{G}_{SM}$ did not itself contain an abelian factor. The existence of a FI term for the $U(1)'$ dynamically generates a similar term for the SM $U(1)_Y$ and this must be accounted for in order to calculate realistic spectra that one can expect to compare with experimental data.

The spectra of sparticles have common features across the parameter space which will be useful in identifying this scenario in collider experiments. These are the light Wino triplet and the existence of relations between the masses of several of the sparticles.

It is possible to consider how this scenario might be compatible with Grand Unified Theories. And example involving the GUT group $SU(5)$ was discussed that resulted in different phenomenology than the original scenario. The spectra here still display the light Wino triplet common to AMSB but are subject to a different set of relations between sparticle masses.

An alternative mechanism for obtaining the electroweak ground state was discussed which does not involve an extended gauge group. Instead a large mass term can be added to the Bino mass parameter M_1. The possibility that M_2 and M_3 also receive a large mass contribution was also explored. These possibilities produce realistic collider scale sparticle spectra with distinctive features that can be used to distinguish these scenarios from those involving the extended gauge group.

AMSB provides an interesting alternative to mSUGRA theories. It is very possible that the physics observed using collider experiments in the near future will resemble these kinds of scenarios. The work presented here will help to identify any of these scenarios which may be discovered and to rule out those which do not describe the new physics accurately. It will also provide a framework for anyone wishing to study related theories.

Appendix A

Two Component Spinors

Many quantum field theorists may be familiar only with the traditional, four-component notation describing Dirac and Majorana spinors. This is described using the 4×4 Dirac matrices γ_μ whose anticommutator must obey the relation $\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}$ where $\eta_{\mu\nu}$ is the spacetime metric. This necessitates the use of left and right projection operators, $P_L = (1 - \gamma_5)/2$ and $P_R = (1 + \gamma_5)/2$ where $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$. A four-component, Dirac fermion with mass m is described by the Lagrangian

$$\mathcal{L}_D = -i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - m\bar{\Psi}\Psi \tag{A.1}$$

Instead of this, many people who deal with supersymmetric theories prefer to use the two-component Weyl notation. This section provides a brief introduction to the two-component formulation to any reader who may be familiar only with four-component spinors.

In the two-component language, the γ-matrices are described in 2×2 blocks,

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix}, \tag{A.2}$$

$$\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{A.3}$$

where

$$\bar{\sigma}_0 = \sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{A.4}$$

$$\bar{\sigma}_1 = -\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{A.5}$$

$$\bar{\sigma}_2 = -\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tag{A.6}$$

$$\bar{\sigma}_3 = -\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.7}$$

In turn, the four-component Dirac spinor is written in terms of two component anti-commuting objects

$$\Psi = \begin{pmatrix} \xi_\alpha \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix}, \tag{A.8}$$

$$\bar{\Psi} = \begin{pmatrix} \chi^\alpha & \xi^\dagger_{\dot{\alpha}} \end{pmatrix}. \tag{A.9}$$

Here, ξ is called the left-handed spinor and $\chi^\dagger$ is the right-handed spinor because

$$P_L\Psi = \begin{pmatrix} \xi_\alpha \\ 0 \end{pmatrix}, \tag{A.10}$$

$$P_R\Psi = \begin{pmatrix} 0 \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix}. \tag{A.11}$$

The hermitian conjugate of any left-handed Weyl spinor is a right-handed Weyl spinor and vice versa. It is conventional to describe the right-handed spinor as the "conjugate field".

Though both the dotted and undotted spinor indices are often omitted, it is important to understand the role they play. The σ-matrices carry such spinor indices thus $(\sigma^\mu)_{\alpha\dot{\alpha}}$, $(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha}$. These indices are raised and lowered using the antisymmetric symbol $\epsilon^{12} = -\epsilon^{21} = \epsilon_{21} = -\epsilon_{12} = 1$, $\epsilon^{11} = \epsilon^{22} = \epsilon_{11} = \epsilon_{22} = 0$ so that

$$\xi_\alpha = \epsilon_{\alpha\beta}\xi^\beta, \tag{A.12}$$

$$\xi^\alpha = \epsilon^{\alpha\beta}\xi_\beta, \tag{A.13}$$

and the same for the conjugate spinors where the antisymmetric symbol obviously carries dotted indices.

When the spinor indices are contracted high to low (or low to high in the case of dotted spinors) they are usually suppressed. For example, $\xi^\dagger_{\dot{\alpha}}(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha}\chi_\alpha$ is usually written simply $\xi^\dagger\bar{\sigma}^\mu\chi$. It is interesting to note that while Weyl spinors are anti-commuting, the object $\xi\chi = \chi\xi$. This is because

$$\xi\chi = \xi^\alpha\chi_\alpha = \xi^\alpha\epsilon_{\alpha\beta}\chi^\beta = -\chi^\beta\epsilon_{\alpha\beta}\xi^\alpha = \chi^\beta\epsilon_{\beta\alpha}\chi^\alpha = \chi^\beta\xi_\beta = \chi\xi. \tag{A.14}$$

.

In this language, the Lagrangian (A.1) is written

$$\mathcal{L}_D = -i\xi^\dagger\bar{\sigma}^\mu\partial_\mu\xi - i\chi^\dagger\bar{\sigma}^\mu\partial_\mu\chi - m(\xi\chi + \xi^\dagger\chi^\dagger). \tag{A.15}$$

For a Majorana spinor, $\xi = \chi$ so that

$$\Psi = \begin{pmatrix} \xi_\alpha \\ \xi^{\dagger\dot{\alpha}} \end{pmatrix}; \tag{A.16}$$

and its Lagrangian can be written

$$\mathcal{L}_M = -i\xi^\dagger\bar{\sigma}^\mu\partial_\mu\xi - \frac{1}{2}m(\xi\xi + \xi^\dagger\xi^\dagger). \tag{A.17}$$

Appendix B

Mass Mixing Matrices

The Lagrangian of the MSSM contains many terms which lead to mixing between the gauge eigenstates of the fields present. Some of these terms are presented here along with the matrices whose eigenvalues are the masses of the physical eigenstates.

The part of the Lagrangian describing the mass terms relating to the gauginos is

$$\mathcal{L} \supset -\frac{1}{2}M_3\bar{\lambda}_a\lambda_a - \frac{1}{2}\bar{\chi}M^0\chi - (\bar{\psi}M^{\pm}\psi + h.c.), \tag{B.1}$$

where the λ_a are the gluino fields and

$$\chi = \begin{pmatrix} \tilde{B}^0 \\ \tilde{W}^3 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \end{pmatrix}, \tag{B.2}$$

$$\psi = \begin{pmatrix} \tilde{W}^{\pm} \\ \tilde{H}^{\pm} \end{pmatrix}. \tag{B.3}$$

The neutralino mass matrix is

$$M^0 = \begin{pmatrix} M_1 & 0 & -M_Z\cos\beta\sin\theta_W & M_Z\sin\beta\sin\theta_W \\ 0 & M_2 & M_Z\cos\beta\cos\theta_W & -M_Z\sin\beta\cos\theta_W \\ -M_Z\cos\beta\sin\theta_W & M_Z\cos\beta\cos\theta_W & 0 & -\mu \\ M_Z\sin\beta\sin\theta_W & -M_Z\sin\beta\cos\theta_W & -\mu & 0 \end{pmatrix}. \tag{B.4}$$

The chargino mass matrix is

$$M^{\pm} = \begin{pmatrix} M_2 & \sqrt{2}M_W\sin\beta \\ \sqrt{2}M_W\cos\beta & \mu \end{pmatrix}. \tag{B.5}$$

For the third generation, where Yukawa couplings are non-negligible, there is a significant mixing between the right and left-handed electroweak eigenstates. The top-squark mixing matrix is

$$\begin{pmatrix} \tilde{m}_{tL}^2 & m_t(A_t - \mu\cot\beta) \\ m_t(A_t - \mu\cot\beta) & \tilde{m}_{tR}^2 \end{pmatrix}. \tag{B.6}$$

For the bottom squarks,

$$\begin{pmatrix} \tilde{m}^2_{bL} & m_b(A_b - \mu\tan\beta) \\ m_b(A_b - \mu\tan\beta) & \tilde{m}^2_{bR} \end{pmatrix}. \tag{B.7}$$

For the tau sleptons

$$\begin{pmatrix} \tilde{m}^2_{\tau L} & m_\tau(A_\tau - \mu\tan\beta) \\ m_\tau(A_\tau - \mu\tan\beta) & \tilde{m}^2_{\tau R} \end{pmatrix}. \tag{B.8}$$

Here the A_i are the trilinear soft terms associated with the relevant field.

The CP-even neutral scalar Higgs $(\text{mass})^2$ mixing matrix is

$$\begin{pmatrix} r_1 - r_2 + \frac{1}{2}M_Z^2(1+2\cos 2\beta) & -r_3 - \frac{1}{2}M_Z^2\sin 2\beta \\ -r_3 - \frac{1}{2}M_Z^2\sin 2\beta & r_1 + r_2 + \frac{1}{2}M_Z^2(1-2\cos 2\beta) \end{pmatrix}, \tag{B.9}$$

where

$$r_1 = \mu^2 + \frac{1}{2}(m_1^2 + m_2^2), \tag{B.10}$$

$$r_2 = \frac{1}{2}(m_2^2 - m_1^2), \tag{B.11}$$

$$r_3 = m_3. \tag{B.12}$$

The CP-odd neutral scalar Higgs $(\text{mass})^2$ mixing matrix is

$$\begin{pmatrix} r_1 - r_2 + \frac{1}{2}M_Z^2(\cos 2\beta) & r_3 \\ r_3 & r_1 + r_2 - \frac{1}{2}M_Z^2(\cos 2\beta) \end{pmatrix}. \tag{B.13}$$

One of the eigenvalues of this matrix is zero, indicating the would-be Goldstone boson that is 'eaten' to become the longitudinal polarisation of the physical Z-boson.

The charged Higgs $(\text{mass})^2$ mixing matrix is

$$\begin{pmatrix} r_1 - r_2 + M_W^2\sin^2\beta + \frac{1}{2}M_Z^2\cos 2\beta & r_3 + \frac{1}{2}M_W^2\sin 2\beta \\ r_3 + \frac{1}{2}M_W^2\sin 2\beta & r_1 + r_2 - \frac{1}{2}M_Z^2\cos 2\beta + M_W^2\cos^2\beta \end{pmatrix}. \tag{B.14}$$

Again there is a would-be Goldstone boson, corresponding to the charged physical W-boson.

Appendix C

Example of Maple code to Calculate Neutralino Masses

The following code, written in Maple, is an example of the programs used to generate the results found in earlier chapters. This calculates the mass of one of the neutralinos in the simplest case. The other scenarios described in this document utilised similar codes to this one, often edited from the earlier versions. The command given in maple is "read Neut1pole;".

An input file can be edited to test different values for the parameters $m_{3/2}$, $\tan\beta$, sign(μ) and the $U(1)'$ charges.

Some of the files which must be read by Maple when running this code were written by D. R. T. Jones and I. Jack, or are edited versions of files originally written by them. These are AMSBrun1new, tadpoles, massesmspec1new, run1alt and MASSPROG1mspec.

```
#To find the pole masses of the neutralinos

read "AMSBrun1new":

musq:=0.6700114805:
parameter1:=1:
mspec:=0.8119436668:
while parameter1>10^(-6) do
read "stuff":
read "neutralino":
parameter1:=evalf(abs(mspec-Neutralino1));
musq:=musqnew:
mspec:=abs(Neutralino1):
od;
MNeut1:=Neutralino1;
```

Here AMSBrun1new determines the running couping values.

C.1 stuff

```
read "neuchinomasses":
read "rot":
read "rotb":
read "Wrot":
read "Zrot":
read "Higgsrot":
```

C.1.1 neuchinomasses

```
read "run1alt":
read "massesmspec1new":
read "tadpoles":

#Quantum numbers 1:electric charge, 2:hypercharge,
#3:third component of isospin
#This is table (A.8) from BPMZ
Quant:=matrix(3,7,[2/3,-2/3,-1/3,1/3,0,-1,1,1/3,-4/3,
1/3,2/3,-1,-1,2,1/2,0,-1/2,0,1/2,-1/2,0]):

#Neutralino couplings to fermions as given by BPMZ

a0uu:=matrix(4,2,0):
a0uu[1,2]:=(gprime/sqrt2)*Quant[2,2]:

a0tt:=matrix(4,2,0):
for i from 1 to 4 do
for j from 1 to 2 do
a0tt[i,j]:=a0uu[i,j]
od: od:
a0tt[4,1]:=lamt:

a0dd:=matrix(4,2,0):
a0dd[1,2]:=(gprime/sqrt2)*Quant[2,4]:

a0bb:=matrix(4,2,0):
for i from 1 to 4 do
```

```
for j from 1 to 2 do
a0bb[i,j]:=a0dd[i,j]
od: od:
a0bb[3,1]:=lamb:

a0ee:=matrix(4,2,0):
a0ee[1,2]:=(gprime/sqrt2)*Quant[2,7]:

a0tau:=matrix(4,2,0):
for i from 1 to 4 do
for j from 1 to 2 do
a0tau[i,j]:=a0ee[i,j]
od: od:
a0tau[3,1]:=lamtau:

b0uu:=matrix(4,2,0):
b0uu[1,1]:=(gprime/sqrt2)*Quant[2,1]:
b0uu[2,1]:=sqrt2*g2*Quant[3,1]:

b0tt:=matrix(4,2,0):
for i from 1 to 4 do
for j from 1 to 2 do
b0tt[i,j]:=b0uu[i,j]
od: od:
b0tt[4,2]:=lamt:

b0dd:=matrix(4,2,0):
b0dd[1,1]:=(gprime/sqrt2)*Quant[2,3]:
b0dd[2,1]:=sqrt2*g2*Quant[3,3]:

b0bb:=matrix(4,2,0):
for i from 1 to 4 do
for j from 1 to 2 do
b0bb[i,j]:=b0dd[i,j]
od: od:
b0bb[3,2]:=lamb:

b0ee:=matrix(4,2,0):
b0ee[1,1]:=(gprime/sqrt2)*Quant[2,6]:
```

```
b0ee[2,1]:=sqrt2*g2*Quant[3,6]:

b0tau:=matrix(4,2,0):
for i from 1 to 4 do
for j from 1 to 2 do
b0tau[i,j]:=b0ee[i,j]
od: od:
b0tau[3,2]:=lamtau:

b0nn:=vector(4,0):
b0nn[1]:=(gprime/sqrt2)*Quant[2,5]:
b0nn[2]:=sqrt2*g2*Quant[3,5]:

#Chargino couplings to fermions as given by BPMZ
apdu:=matrix(2,2,0):
apdu[1,1]:=g2:

apdt:=apdu:
apbu:=apdu:

apbt:=matrix(2,2,0):
for i from 1 to 2 do
for j from 1 to 2 do
apbt[i,j]:=apdu[i,j]
od: od:
apbt[2,2]:=-lamt:

apud:=matrix(2,2,0):

apub:=apud:
aptd:=apud:

aptb:=matrix(2,2,0):
for i from 1 to 2 do
for j from 1 to 2 do
aptb[i,j]:=apud[i,j]
od: od:
aptb[2,1]:=-lamt:
```

```
bpud:=matrix(2,2,0):
bpud[1,1]:=g2:

bpub:=bpud:
bptd:=bpud:

bptb:=matrix(2,2,0):
for i from 1 to 2 do
for j from 1 to 2 do
bptb[i,j]:=bpub[i,j]
od: od:
bptb[2,2]:=-lamb:

bpdu:=matrix(2,2,0):

bpdt:=bpdu:
bpbu:=bpdu:

bpbt:=matrix(2,2,0):
for i from 1 to 2 do
for j from 1 to 2 do
bpbt[i,j]:=bpbu[i,j]
od: od:
bpbt[2,1]:=-lamb:

apenu:=matrix(2,2,0):
apenu[1,1]:=g2:

aptaunu:=apenu:
apenutau:=apenu:

aptaunutau:=matrix(2,2,0):
for i from 1 to 2 do
for j from 1 to 2 do
aptaunutau[i,j]:=apenu[i,j]
od: od:

apnue:=matrix(2,2,0):
```

```
apnutaue:=apnue:
apnutau:=apnue:

apnutautau:=matrix(2,2,0):
for i from 1 to 2 do
for j from 1 to 2 do
apnutautau[i,j]:=apnue[i,j]
od: od:

bpenu:=matrix(2,2,0):
bptaunu:=bpenu:
bpenutau:bpenu:

bptaunutau:=matrix(2,2,0):
for i from 1 to 2 do
for j from 1 to 2 do
bptaunutau[i,j]:=bpenu[i,j]
od: od:
bptaunutau[2,1]:=-lamtau:

bpnue:=matrix(2,2,0):
bpnue[1,1]:=g2:

bpnutaue:=bpnue:

bpnutau:=bpnue:

bpnutautau:=matrix(2,2,0):
for i from 1 to 2 do
for j from 1 to 2 do
bpnutautau[i,j]:=bpnue[i,j]
od: od:
bpnutautau[2,2]:=-lamtau:

#Couplings to Zs
a00Z:=matrix(4,4,0):
a00Z[3,3]:=g2/(2*cos(thetaw)):
a00Z[4,4]:=-g2/(2*cos(thetaw)):
```

```
b00Z:=matrix(4,4,0):
for i from 1 to 4 do
for j from 1 to 4 do
b00Z[i,j]:=(-1)*a00Z[i,j]
od: od:

appZ:=matrix(2,2,0):
appZ[1,1]:=g2*cos(thetaw):
appZ[2,2]:=g2*cos(2*thetaw)/(2*cos(thetaw)):

bppZ:=appZ:

#Couplings to Ws
a0pW:=matrix(4,2,0):
a0pW[2,1]:=-g2:
a0pW[4,2]:=g2/sqrt2:

b0pW:=matrix(4,2,0):
b0pW[2,1]:=-g2:
b0pW[3,2]:=-g2/sqrt2:


#Chargino couplings to photons
appga:=matrix(2,2,0):
appga[1,1]:=sqrt(4*Pi1*alphem):
appga[2,2]:=sqrt(4*Pi1*alphem):

bppga:=appga:

read "Higgscouple":


topmix:=matrix(2,2,[cos(thetat),sin(thetat),-sin(thetat),cos(thetat)]):
botmix:=matrix(2,2,[cos(thetab),sin(thetab),-sin(thetab),cos(thetab)]):
taumix:=matrix(2,2,[cos(thetatau),sin(thetatau),-sin(thetatau),
cos(thetatau)]):
```

run1alt

```
#one loop running only
```

```
tspec:= ln(mspec/.09119)/(2*Pi):
tfinald:=tmax:
tspecd:=tmax - tspec:
sys1:={diff(a3(t),t) =  betag3,
diff(a2(t),t) = betag2,
diff(a1(t),t) = betag1,
diff(yt(t),t) =  betalt1,
diff(yb(t),t) =  betalb1,
diff(ytau(t),t) =  betaltau1,
diff(v1(t),t) = gah1*v1(t)/2,
diff(v2(t),t) = gah2*v2(t)/2,
a3(0) = za3, a2(0) = za2, a1(0) = za1, yt(0) = ytz,
yb(0) = ybz, ytau(0) = ytauz, v1(0) = v1z, v2(0) = v2z};
fcns := {a3(t),a2(t),a1(t),yt(t),yb(t),ytau(t), v1(t), v2(t)};
ff:= dsolve(sys1, fcns, numeric, output=listprocedure);
alph3 := subs(ff,a3(t));
alph2 := subs(ff,a2(t));
alph1 := subs(ff,a1(t));
yukt := subs(ff,yt(t));
yukb := subs(ff,yb(t));
yuktau := subs(ff,ytau(t));
vev1:= subs(ff,v1(t));
vev2:= subs(ff,v2(t));
#OUTPUTS at MX
ytX:=yukt(tmax);
ybX:=yukb(tmax);
ytauX:=yuktau(tmax);
a3X:=alph3(tmax);
a2X:=alph2(tmax);
a1X:=alph1(tmax);
#outputs at spectrum scale
ytspec:=yukt(tspec);
ybspec:=yukb(tspec);
ytauspec:=yuktau(tspec);
a3spec:=alph3(tspec);
a2spec:=alph2(tspec);
g2spec:=evalf(sqrt(4*Pi*a2spec));
a1spec:=alph1(tspec);
v1spec:=vev1(tspec);
```

```
v2spec:=vev2(tspec);
a1prime:=a1spec*3/5;
alphem:=a2spec*a1prime/(a2spec+a1prime);
```

massesmspec1new

```
#massesmspec
with(linalg):
v1s:=v1spec;
v2s:=v2spec;
a1prime:=a1spec*3/5;
g2:=sqrt(4*Pi1*a2spec);
g1:=sqrt(4*Pi1*a1spec);
gprime:=sqrt(4*Pi1*a1prime);
alphem:=a2spec*a1prime/(a2spec+a1prime);
tanbet:=v2s/v1s;
bet:=arctan(tanbet);
cosbet:=cos(bet);
sinbet:=sin(bet);
cotbet:=1/tanbet;
costbet:=cos(2*bet);
v:= sqrt(v1s^2+v2s^2);
MW := g2*v/2;
MZ := sqrt(g2^2+gprime^2)*v/2;
MW2 :=(MW)^2;
MZ2:=(MZ)^2;
costhetaw:= MW/MZ;
thetaw:=arccos(costhetaw);
##neutral current couplings
gtL:=0.5-(2/3)*sin(thetaw)^2;
gtR:=(2/3)*sin(thetaw)^2:
gbL:=-0.5+(1/3)*sin(thetaw)^2:#sign changed Feb 2
gbR:=-(1/3)*sin(thetaw)^2: #sign changed Feb 2
gtauL:= -0.5+sin(thetaw)^2;
gtauR:= -sin(thetaw)^2;
gnu:=0.5;
lamt:=sqrt(4*Pi1*ytspec);
lamb:=sqrt(4*Pi1*ybspec);
lamtau:=sqrt(4*Pi1*ytauspec);
Yt:=evalm(matrix(3,3,[0,0,0,0,0,0,t31,t32,t33])*lamt):
```

```
Yb:=evalm(matrix(3,3,[0,0,0,0,0,0,bot31,bot32,bot33])*lamb):
Ytau:=evalm(matrix(3,3,[0,0,0,0,0,0,tau31,tau32,tau33])*lamtau):
mtop:= Yt*v2s/sqrt2:
mbot:=Yb*v1s/sqrt2:
masstau:= Ytau*v1s/sqrt2:
mtopT:= transpose(Yt*v2s/sqrt2):
mbotT:= transpose(Yb*v1s/sqrt2):
masstauT:= transpose(Ytau*v1s/sqrt2):
YtT:=transpose(Yt):
YbT:=transpose(Yb):
YtauT:=transpose(Ytau):
read MASSPROG1mspec;
ht:= m10spec*Yt:
hb:= m8spec*Yb:
htau:= m6spec*Ytau:
mmq:=evalm(matrix(3,3,[mrspec,0,0,0,mrspec,0,0,0, mqspec]));
mmt:=evalm(matrix(3,3,[mupspec,0,0,0,mupspec,0,0,0, mtspec]));
mmb:=evalm(matrix(3,3,[mdspec,0,0,0,mdspec,0,0,0, mbspec]));
mmL:=evalm(matrix(3,3,[mnspec,0,0,0,mnspec,0,0,0, mLspec]));
mmtau:= evalm(matrix(3,3,[mespec,0,0,0,mespec,0,0,0, mtauspec]));
mmh1:=m1spec;
mmh2:=m2spec;
#Higgs potential minimisation
musqtree:=evalf((mmh1-mmh2*tanbet^2)/(tanbet^2-1)-0.5*MZ2);
muterm:= evalf(signmu*sqrt(musq));
mutermtree:=evalf(signmu*sqrt(musqtree));
MUTERMB:=-muterm;
r2:=0.5*(mmh2-mmh1);
r1:=0.5*(mmh2+mmh1)+ musqtree;
r3:= r1*sin(2*bet);
#the 6 by 6 mass matrices;
mstop11:= evalm( mmq+multiply(mtopT,mtop) +(4*MW2 - MZ2)*cos(2*bet)/6);
mstop21:= evalm(evalf(ht*v2s/sqrt2-muterm*cotbet*mtop));
mstop12:= evalm(transpose(mstop21));
mstop22:= evalm( mmt+ multiply(mtop,mtopT) - 2*(MW2 - MZ2)*costbet/3);
mstop:= blockmatrix(2,2,[mstop11,mstop12,mstop21,mstop22]);
msbot11:= evalm( mmq+ multiply(mbotT,mbot)-(2*MW2 +MZ2)*costbet/6);
msbot21:= evalm(hb*v1s/sqrt2-muterm*tan(bet)*mbot);
msbot12:=transpose(msbot21);
```

```
msbot22:= evalm( mmb+multiply(mbot,mbotT) + (MW2 - MZ2)*costbet/3);
msbot:= blockmatrix(2,2,[msbot11,msbot12,msbot21,msbot22]);
mstau11:= evalm( mmL+ multiply(masstauT,masstau) -
(2*MW2 - MZ2)*costbet/2);
mstau21:= evalm( htau*v1s/sqrt2-muterm*tan(bet)*masstau);
mstau12:=transpose(mstau21);
mstau22:= evalm( mmtau+multiply(masstau,masstauT)+
(MW2 - MZ2)*costbet);
mstau:= evalm(blockmatrix(2,2,[mstau11,mstau12,mstau21,mstau22]));
msnu:= evalm(mmL+0.5*MZ2*cos(2*bet));
mslino:= matrix(4,4);
mslino[1,1]:= M1spec:
mslino[1,2]:= 0:
mslino[2,1]:= 0:
mslino[3,3]:= 0:
mslino[4,4]:= 0:
mslino[2,2]:= M2spec;
mslino[3,4]:= -muterm:
mslino[4,3]:= mslino[3,4];
mslino[1,3]:= -MZ*cos(bet)*sin(thetaw);
mslino[3,1]:=mslino[1,3];
mslino[1,4]:= MZ*sin(bet)*sin(thetaw):
mslino[4,1]:=mslino[1,4];
mslino[2,3]:= MZ*cos(bet)*cos(thetaw);
mslino[3,2]:=mslino[2,3]:
mslino[2,4]:= -MZ*sin(bet)*cos(thetaw);
mslino[4,2]:= mslino[2,4];
mschino:= array(1..2, 1..2);
mschino[1,1]:= M2spec;
mschino[1,2]:= sqrt2*MW*sin(bet);
mschino[2,1]:= sqrt2*MW*cos(bet);
mschino[2,2]:= muterm:
mshiggseven:= array(1..2, 1..2);
mshiggseven[1,1]:= evalf(r1 - r2 +0.5*MZ2*(1 + 2*costbet));
mshiggseven[1,2]:= evalf(-r3 - 0.5*MZ2*sin(2*bet));
mshiggseven[2,1]:= mshiggseven[1,2];
mshiggseven[2,2]:= evalf(r1 + r2 +0.5*MZ2*(1 - 2*costbet));
mshiggsodd:= array(1..2, 1..2);
mshiggsodd[1,1]:= r1 - r2 +0.5*MZ2*costbet;
```

```
mshiggsodd[1,2]:= r3;
mshiggsodd[2,1]:= mshiggsodd[1,2]:
mshiggsodd[2,2]:= r1 + r2 - 0.5*MZ2*costbet;
mshiggschgd:= array(1..2, 1..2);
mshiggschgd[1,1]:= r1 - r2 + MW2*(sin(bet))^2 + 0.5*MZ2*costbet;
mshiggschgd[1,2]:= r3 + 0.5*MW2*sin(2*bet);
mshiggschgd[2,1]:= mshiggschgd[1,2];
mshiggschgd[2,2]:= r1 + r2 + MW2*(cos(bet))^2 - 0.5*MZ2*costbet;
mstopmasses:= eigenvals(mstop);
msbotmasses:= eigenvals(msbot);
mstaumasses:= eigenvals(mstau);
mshgevmasses:=  eigenvals(mshiggseven);
mshgoddmasses:=  eigenvals(mshiggsodd);
mshgchgdmasses:=  eigenvals(mshiggschgd):
mst1 :=evalf(sqrt(mstopmasses[1]));
mst2 :=evalf(sqrt(mstopmasses[2]));
mst3 :=evalf(sqrt(mstopmasses[3]));
mst4 :=evalf(sqrt(mstopmasses[4]));
mst5 :=evalf(sqrt(mstopmasses[5]));
mst6 :=evalf(sqrt(mstopmasses[6]));
msb1 :=evalf(sqrt(msbotmasses[1]));
msb2 :=evalf(sqrt(msbotmasses[2])):
msb3 :=evalf(sqrt(msbotmasses[3]));
msb4 :=evalf(sqrt(msbotmasses[4]));
msb5 :=evalf(sqrt(msbotmasses[5]));
msb6 :=evalf(sqrt(msbotmasses[6]));
msqav :=(mst1 +mst2 +mst3 +mst4 +mst5 +mst6 +
msb1 +msb2 +msb3 +msb4 +msb5 +msb6 )/12;
mstau1 :=evalf(sqrt(mstaumasses[1]));
mstau2 :=evalf(sqrt(mstaumasses[2]));
mstau3 :=evalf(sqrt(mstaumasses[3]));
mstau4 :=evalf(sqrt(mstaumasses[4]));
mstau5 :=evalf(sqrt(mstaumasses[5]));
mstau6 :=evalf(sqrt(mstaumasses[6]));
msLav :=(mstau1 +mstau2 +mstau3
+mstau4 +mstau5 +mstau6 )/6;
mhggs1:=evalf(sqrt(abs(mshgevmasses[1])));
mhggs2:=evalf(sqrt(abs(mshgevmasses[2])));
mhiggs1:=min(mhggs1,mhggs2);
```

```
mhiggs2:=max(mhggs1,mhggs2);
mgg1:=sqrt(abs(mshgoddmasses[1]));
mgg2:=sqrt(abs(mshgoddmasses[2]));
mA0:= max(mgg1,mgg2);
mgp1:=sqrt(abs(mshgchgdmasses[1]));
mgp2:=sqrt(abs(mshgchgdmasses[2]));
mhplus:=max(mgp1,mgp2);
mslinomasses:=evalf( eigenvals(mslino));
# create a slinodiag with all positive eigenvalues
evalf(Svd(mslino,Uslino,Vslino));
slinodiagcheck:= evalm(transpose(Uslino)&*mslino&*Uslino);
if slinodiagcheck[1,1] < 0
then Nex1:= I else Nex1:=1 end if;
if slinodiagcheck[2,2] < 0
then Nex2:= I else Nex2:=1 end if;
if slinodiagcheck[3,3] < 0
then Nex3:= I else Nex3:=1 end if;
if slinodiagcheck[4,4] < 0
then Nex4:= I else Nex4:=1 end if;
Nextra:=matrix(4,4, [Nex1,0,0,0,0,Nex2,0,0,0,0,Nex3,0, 0,0,0,Nex4]);
Uslinox:=evalm(Uslino&*Nextra);
slinodiag:= evalm(transpose(Uslinox)&*mslino&*Uslinox);
Nslino:=htranspose(Uslinox);
slino1:=slinodiag[1,1];
slino2:=slinodiag[2,2];
slino3:=slinodiag[3,3];
slino4:=slinodiag[4,4];
#chargino diagonalisation
evalf(Svd(mschino,Uchino,Vchino));
chinodiag:= evalm(htranspose(Uchino)&*mschino&*Vchino);
Vch:=htranspose(Vchino);
Uch:=transpose(Uchino);
#mschinosqrd:=evalm(mschino&*transpose(mschino));
#mschinomasses:=eigenvals(mschinosqrd);
#mschino1 :=evalf(sqrt(mschinomasses[1]));
#mschino2 :=evalf(sqrt(mschinomasses[2]));
chino1:=chinodiag[1,1];
chino2:=chinodiag[2,2];
snutau:=msnu[3,3];
```

```
snumu:=msnu[1,1];
msnutau :=evalf(sqrt(snutau));
msnumu :=evalf(sqrt(snumu));
mtopspec:=evalf(sqrt(2*Pi1*ytspec)*v2s);
mbotspec:=evalf(sqrt(2*Pi1*ybspec)*v1s);
mtaulepspec:=evalf(sqrt(2*Pi1*ytauspec)*v1s);

thetat:= 0.5*arctan(2*mstop[3,6]/(mstop[3,3]-mstop[6,6]));
stop1:=mstop[3,3]*cos(thetat)^2+mstop[6,6]*sin(thetat)^2
+sin(2*thetat)*mstop[3,6];
stop2:=mstop[3,3]*sin(thetat)^2+mstop[6,6]*cos(thetat)^2
-sin(2*thetat)*mstop[3,6];
mup1:=mstop[1,1];
mup2:=mstop[4,4];
md1:=msbot[1,1];
md2:=msbot[4,4];

smu1:=mstau[1,1];
smu2:=mstau[4,4];
thetab:= 0.5*arctan(2*msbot[3,6]/(msbot[3,3]-msbot[6,6]));
sbot1:=msbot[3,3]*cos(thetab)^2+msbot[6,6]*sin(thetab)^2
+sin(2*thetab)*msbot[3,6];
sbot2:=msbot[3,3]*sin(thetab)^2+msbot[6,6]*cos(thetab)^2
-sin(2*thetab)*msbot[3,6];
thetatau:= 0.5*arctan(2*mstau[3,6]/(mstau[3,3]-mstau[6,6]));
stau1:=mstau[3,3]*cos(thetatau)^2+mstau[6,6]*sin(thetatau)^2
+sin(2*thetatau)*mstau[3,6];
stau2:=mstau[3,3]*sin(thetatau)^2+mstau[6,6]*cos(thetatau)^2
-sin(2*thetatau)*mstau[3,6];

##higgs mass stuff from polonski review
deltamh1:=evalf(3*alphem*(mtopspec)^4*(ln(stop1*stop2/mtopspec^4))
/(4*Pi1*(sin(thetaw))^2*(cos(thetaw))^2*MZ2));
deltat1:= (stop1-stop2)*(sin(2*thetat))^2*
ln(stop1/stop2)/(2*mtopspec^2);
deltat2:= ((stop1-stop2)*(sin(2*thetat))^2/(4*mtopspec^2))^2
*(2- (stop1+stop2)*ln(stop1/stop2)/
(stop1-stop2));
deltat:=deltat1+deltat2;
```

```
deltamh2:=evalf(3*alphem*(mtopspec)^4*deltat
/(4*Pi1*(sin(thetaw))^2*(cos(thetaw))^2*MZ2));

Higgs := evalf(sqrt(mhiggs1^2 + deltamh1+deltamh2));

mt1 :=sqrt(stop1);#tree values
mt2 :=sqrt(stop2);
mb1 :=sqrt(sbot1);#TREE values
mb2 :=sqrt(sbot2);
alphH:=0.5*arctan((mA0^2+MZ2)/(mA0^2-MZ2)*tan(2*bet));
MA2:=mA0^2;
```

MASSPROG1mspec

```
atmspec:=x -> subs(a1n=a1spec,a2n=a2spec,a3n=a3spec,
ytn=ytspec,ybn=ybspec,ytaun=ytauspec,x);
mqspec:=atmspec(mzero^2/(16*Pi1^2)*massdiff1(gaqn))+xx1*hypq;
mtspec:=atmspec(mzero^2/(16*Pi1^2)*massdiff1(gatn))+xx1*hypt;
mbspec:=atmspec(mzero^2/(16*Pi1^2)*massdiff1(gabn))+xx1*hypb;
mLspec:=atmspec(mzero^2/(16*Pi1^2)*massdiff1(gaLn))+xx1*hypL;
mtauspec:=atmspec(mzero^2/(16*Pi1^2)*massdiff1(gataun))+xx1*hyptau;
mrspec:=atmspec(mzero^2/(16*Pi1^2)*massdiff1(garn))+xx1*hypq;
mdspec:=atmspec(mzero^2/(16*Pi1^2)*massdiff1(gadn))+xx1*hypb;
mnspec:=atmspec(mzero^2/(16*Pi1^2)*massdiff1(gann))+xx1*hypL;
mespec:=atmspec(mzero^2/(16*Pi1^2)*massdiff1(gaen))+xx1*hyptau;
mupspec:=atmspec(mzero^2/(16*Pi1^2)*massdiff1(gaun))+xx1*hypt;
m1spec:=atmspec(mzero^2/(16*Pi1^2)*massdiff1(gah1n))+xx1*hyph1;
m2spec:=atmspec(mzero^2/(16*Pi1^2)*massdiff1(gah2n))+xx1*hyph2;
m10spec:=-mzero*(atmspec(gatn)+atmspec(gaqn)+atmspec(gah2n))/4/Pi1;
m8spec:=-mzero*(atmspec(gabn)+atmspec(gaqn)+atmspec(gah1n))/4/Pi1;
m6spec:=-mzero*(atmspec(gataun)+atmspec(gaLn)+atmspec(gah1n))/4/Pi1;
M1spec:= mzero*atmspec(betag1n/a1n)/4/Pi1;
M2spec:= mzero*atmspec(betag2n/a2n)/4/Pi1;
M3spec:= mzero*atmspec(betag3n/a3n)/4/Pi1;
```

tadpoles

```
#tadpoles

#First bit exactly as it looks
A0stop1:=A0(stop1);
```

```
A0stop2:=A0(stop2);
A0sbot1:=A0(sbot1);
A0sbot2:=A0(sbot2);
A0sup1:=A0(mup1);
A0sup2:=A0(mup2);
A0sdown1:=A0(md1);
A0sdown2:=A0(md2);
A0stau1:=A0(stau1);
A0stau2:=A0(stau2);
A0smuon1:=A0(me1);
A0smuon2:=A0(me2);
A0snutau:=A0(msnu[3,3]);
A0snumu:=A0(msnu[2,2]);
A0mA:=A0(mA0^2);
A0mhplus:=A0(mhplus^2);
A0mh:=A0(mhiggs1^2);
A0mH:=A0(mhiggs2^2);
A0slino1:=A0(slino1^2);
A0slino2:=A0(slino2^2);
A0slino3:=A0(slino3^2);
A0slino4:=A0(slino4^2);
A0chino1:=A0(chino1^2);
A0chino2:=A0(chino2^2);
A0MW:=A0(MW2);
A0MZ:=A0(MZ2);

#these defs used to be in stopexact2 and sbotexact2
Ls1tLtL:=g2*MZ*gtL*cos(bet)/cos(thetaw);
Ls1tRtR:=g2*MZ*gtR*cos(bet)/cos(thetaw);
Ls1tLtR:=lamt*MUTERMB/sqrt(2);
Ls1tRtL:=Ls1tLtR;
Ls2tLtL:=-g2*MZ*gtL*sin(bet)/cos(thetaw)+lamt*mtopspec*sqrt(2) ;
Ls2tRtR:=-g2*MZ*gtR*sin(bet)/cos(thetaw)+lamt*mtopspec*sqrt(2);
Ls2tLtR:=lamt*m10spec/sqrt(2);
Ls2tRtL:=Ls2tLtR;
Ls1tLt1:=Ls1tLtL*cos(thetat)+Ls1tLtR*sin(thetat);
Ls1tLt2:=-Ls1tLtL*sin(thetat)+Ls1tLtR*cos(thetat);
Ls1tRt1:=Ls1tRtL*cos(thetat)+Ls1tRtR*sin(thetat);
Ls1tRt2:=-Ls1tRtL*sin(thetat)+Ls1tRtR*cos(thetat);
```

```
Ls2tLt1:=Ls2tLtL*cos(thetat)+Ls2tLtR*sin(thetat);
Ls2tLt2:=-Ls2tLtL*sin(thetat)+Ls2tLtR*cos(thetat);
Ls2tRt1:=Ls2tRtL*cos(thetat)+Ls2tRtR*sin(thetat);
Ls2tRt2:=-Ls2tRtL*sin(thetat)+Ls2tRtR*cos(thetat);

Ls1bLbL:=g2*MZ*gbL*cos(bet)/cos(thetaw)+lamb*mbotspec*sqrt(2) ;
Ls1bRbR:=g2*MZ*gbR*cos(bet)/cos(thetaw)+lamb*mbotspec*sqrt(2);
Ls1bLbR:=lamb*m8spec/sqrt(2);
Ls1bRbL:= Ls1bLbR;
Ls2bLbL:=-g2*MZ*gbL*sin(bet)/cos(thetaw);
Ls2bRbR:=-g2*MZ*gbR*sin(bet)/cos(thetaw);
Ls2bLbR:= lamb*MUTERMB/sqrt(2);
Ls2bRbL:= Ls2bLbR;
Ls1bLb1:=Ls1bLbL*cos(thetab)+Ls1bLbR*sin(thetab);
Ls1bLb2:=-Ls1bLbL*sin(thetab)+Ls1bLbR*cos(thetab);
Ls1bRb1:=Ls1bRbL*cos(thetab)+Ls1bRbR*sin(thetab);
Ls1bRb2:=-Ls1bRbL*sin(thetab)+Ls1bRbR*cos(thetab);
Ls2bLb1:=Ls2bLbL*cos(thetab)+Ls2bLbR*sin(thetab);
Ls2bLb2:=-Ls2bLbL*sin(thetab)+Ls2bLbR*cos(thetab);
Ls2bRb1:=Ls2bRbL*cos(thetab)+Ls2bRbR*sin(thetab);
Ls2bRb2:=-Ls2bRbL*sin(thetab)+Ls2bRbR*cos(thetab);

Ls1tauLtauL:=g2*MZ*gtauL*cos(bet)/cos(thetaw)
+lamtau*mtaulepspec*sqrt(2) ;
Ls1tauRtauR:=g2*MZ*gtauR*cos(bet)/cos(thetaw)
+lamtau*mtaulepspec*sqrt(2);
Ls1tauLtauR:=lamtau*m6spec/sqrt(2);
Ls1tauRtauL:= Ls1tauLtauR;
Ls2tauLtauL:=-g2*MZ*gtauL*sin(bet)/cos(thetaw);
Ls2tauRtauR:=-g2*MZ*gtauR*sin(bet)/cos(thetaw);
Ls2tauLtauR:= lamtau*MUTERMB/sqrt(2);
Ls2tauRtauL:= Ls2tauLtauR;
Ls1tauLtau1:=Ls1tauLtauL*cos(thetatau)+Ls1tauLtauR*sin(thetatau);
Ls1tauLtau2:=-Ls1tauLtauL*sin(thetatau)+Ls1tauLtauR*cos(thetatau);
Ls1tauRtau1:=Ls1tauRtauL*cos(thetatau)+Ls1tauRtauR*sin(thetatau);
Ls1tauRtau2:=-Ls1tauRtauL*sin(thetatau)+Ls1tauRtauR*cos(thetatau);
Ls2tauLtau1:=Ls2tauLtauL*cos(thetatau)+Ls2tauLtauR*sin(thetatau);
Ls2tauLtau2:=-Ls2tauLtauL*sin(thetatau)+Ls2tauLtauR*cos(thetatau);
Ls2tauRtau1:=Ls2tauRtauL*cos(thetatau)+Ls2tauRtauR*sin(thetatau);
```

```
Ls2tauRtau2:=-Ls2tauRtauL*sin(thetatau)+Ls2tauRtauR*cos(thetatau);



Ls1uLuL:=g2*MZ*gtL*cos(bet)/cos(thetaw);
Ls1uRuR:=g2*MZ*gtR*cos(bet)/cos(thetaw);
Ls1uLuR:=0;
Ls1uRuL:=Ls1uLuR;
Ls2uLuL:=-g2*MZ*gtL*sin(bet)/cos(thetaw);
Ls2uRuR:=-g2*MZ*gtR*sin(bet)/cos(thetaw);

Ls1dLdL:=g2*MZ*gbL*cos(bet)/cos(thetaw);
Ls1dRdR:=g2*MZ*gbR*cos(bet)/cos(thetaw);
Ls2dLdL:=-g2*MZ*gbL*sin(bet)/cos(thetaw);
Ls2dRdR:=-g2*MZ*gbR*sin(bet)/cos(thetaw);

Ls1muLmuL:=g2*MZ*gtauL*cos(bet)/cos(thetaw);
Ls1muRmuR:=g2*MZ*gtauR*cos(bet)/cos(thetaw);
Ls2muLmuL:=-g2*MZ*gtauL*sin(bet)/cos(thetaw);
Ls2muRmuR:=-g2*MZ*gtauR*sin(bet)/cos(thetaw);

Ls1nunu:= g2*MZ*gnu*cos(bet)/cos(thetaw);
Ls2nunu:=-g2*MZ*gnu*sin(bet)/cos(thetaw);



Ls1t1t1:=cos(thetat)*Ls1tLt1+sin(thetat)*Ls1tRt1;
Ls1t2t2:= -sin(thetat)*Ls1tLt2 + cos(thetat)*Ls1tRt2;
Ls2t1t1:=cos(thetat)*Ls2tLt1+sin(thetat)*Ls2tRt1;
Ls2t2t2:= -sin(thetat)*Ls2tLt2 + cos(thetat)*Ls2tRt2;
Ls1b1b1:=cos(thetab)*Ls1bLb1+sin(thetab)*Ls1bRb1;
Ls1b2b2:= -sin(thetab)*Ls1bLb2 + cos(thetab)*Ls1bRb2;
Ls2b1b1:=cos(thetab)*Ls2bLb1+sin(thetab)*Ls2bRb1;
Ls2b2b2:= -sin(thetab)*Ls2bLb2 + cos(thetab)*Ls2bRb2;
Ls1tau1tau1:=cos(thetatau)*Ls1tauLtau1+sin(thetatau)*Ls1tauRtau1;
Ls1tau2tau2:= -sin(thetatau)*Ls1tauLtau2
+ cos(thetatau)*Ls1tauRtau2;
Ls2tau1tau1:=cos(thetatau)*Ls2tauLtau1+sin(thetatau)*Ls2tauRtau1;
Ls2tau2tau2:= -sin(thetatau)*Ls2tauLtau2
+ cos(thetatau)*Ls2tauRtau2;
```

```
tad11:=-2*(3*ybspec*A0(mbotspec^2)+ ytauspec*A0(mtaulepspec^2));
tad12:= evalf((1/(4*Pi))*g2/2/MW/cos(bet)*(3*(
Ls1t1t1*A0stop1+Ls1t2t2*A0stop2+Ls1b1b1*A0sbot1+Ls1b2b2*A0sbot2+
2*(Ls1uLuL*A0sup1+Ls1uRuR*A0sup2+Ls1dLdL*A0sdown1+Ls1dRdR*A0sdown2))
+Ls1tau1tau1*A0stau1+Ls1tau2tau2*A0stau2
+2*(Ls1muLmuL*A0smuon1+Ls1muRmuR*A0smuon2)
+Ls1nunu*(A0snutau+2*A0snumu) ));
tad13a:=-a2spec*cos(2*bet)/8/cos(thetaw)^2*(A0mA+2*A0mhplus)
+a2spec/2*A0mhplus;
tad13b:=a2spec/8/cos(thetaw)^2*(
(3*sin(alphH)^2-cos(alphH)^2+sin(2*alphH)*tan(bet))*A0mh
+(3*cos(alphH)^2-sin(alphH)^2-sin(2*alphH)*tan(bet))*A0mH);
tad14:=-a2spec/MW/cos(bet)*Re(
slino1*A0slino1*Nslino[1,3]*(Nslino[1,2]-Nslino[1,1]*tan(thetaw))+
slino2*A0slino2*Nslino[2,3]*(Nslino[2,2]-Nslino[2,1]*tan(thetaw))+
slino3*A0slino3*Nslino[3,3]*(Nslino[3,2]-Nslino[3,1]*tan(thetaw))+
slino4*A0slino4*Nslino[4,3]*(Nslino[4,2]-Nslino[4,1]*tan(thetaw)));
tad15:=evalf(-sqrt(2)*a2spec/MW/cos(bet)*Re(
chino1*A0chino1*(Vch[1,1]*Uch[1,2])
+chino2*A0chino2*(Vch[2,1]*Uch[2,2])));
tad16a:=3*a2spec/4*(2*A0MW+A0MZ/cos(thetat)^2);
tad16b:=a2spec*cos(2*bet)/8/cos(thetat)^2*(2*A0MW+A0MZ);
tad1:=evalf((tad11+tad12+tad13a+tad13b
+tad14+tad15+tad16a+tad16b)/4/Pi);#omit v1 factor

tad21:=-2*(3*ytspec*A0(mtopspec^2));
tad22:= evalf((1/(4*Pi))*g2/2/MW/sin(bet)*(3*(
Ls2t1t1*A0stop1+Ls2t2t2*A0stop2+Ls2b1b1*A0sbot1+Ls2b2b2*A0sbot2+
2*(Ls2uLuL*A0sup1+Ls2uRuR*A0sup2+Ls2dLdL*A0sdown1+Ls2dRdR*A0sdown2))
+Ls2tau1tau1*A0stau1+Ls2tau2tau2*A0stau2
+2*(Ls2muLmuL*A0smuon1+Ls2muRmuR*A0smuon2)
+ Ls2nunu*(A0snutau+2*A0snumu) ));
tad23:= a2spec*cos(2*bet)/8/cos(thetaw)^2*(A0mA+2*A0mhplus)
+a2spec/2*A0mhplus + a2spec/8/cos(thetaw)^2*(
(3*cos(alphH)^2-sin(alphH)^2+sin(2*alphH)*cot(bet))*A0mh
+(3*sin(alphH)^2-cos(alphH)^2-sin(2*alphH)*cot(bet))*A0mH);
```

```
tad24:=+a2spec/MW/sin(bet)*Re(
slino1*A0slino1*Nslino[1,4]*(Nslino[1,2]-Nslino[1,1]*tan(thetaw))+
slino2*A0slino2*Nslino[2,4]*(Nslino[2,2]-Nslino[2,1]*tan(thetaw))+
slino3*A0slino3*Nslino[3,4]*(Nslino[3,2]-Nslino[3,1]*tan(thetaw))+
slino4*A0slino4*Nslino[4,4]*(Nslino[4,2]-Nslino[4,1]*tan(thetaw)));
tad25:=evalf(-sqrt(2)*a2spec/MW/sin(bet)*Re(
chino1*A0chino1*(Vch[1,2]*Uch[1,1])
+chino2*A0chino2*(Vch[2,2]*Uch[2,1])));
tad2:=evalf((tad21+tad22+tad23+tad24+tad25+tad16a-tad16b)/4/Pi);
#omit v2 factor



mmh1new:=mmh1-tad1;
mmh2new:=mmh2-tad2;
musqnew:=evalf((mmh1new-mmh2new*tanbet^2)/(tanbet^2-1)-0.5*MZ2);
```

Higgscouple

```
a00s:=array(1..4,1..4,1..2):
for i from 1 to 4 do
for j from 1 to 4 do
for k from 1 to 2 do
a00s[i,j,k]:=0
od: od: od:
a00s[1,3,1]:=-gprime/2:
a00s[1,4,2]:=gprime/2:
a00s[2,3,1]:=g2/2:
a00s[2,4,2]:=-g2/2:
a00s[3,1,1]:=a00s[1,3,1]:
a00s[4,1,2]:=a00s[1,4,2]:
a00s[3,2,1]:=a00s[2,3,1]:
a00s[4,2,2]:=a00s[2,4,2]:

a00p:=array(1..4,1..4,1..2):
for i from 1 to 4 do
for j from 1 to 4 do
for k from 1 to 2 do
a00p[i,j,k]:=0
od: od: od:
```

```
a00p[1,3,1]:=-gprime/2:
a00p[1,4,2]:=-gprime/2:
a00p[2,3,1]:=g2/2:
a00p[2,4,2]:=g2/2:
a00p[3,1,1]:=a00p[1,3,1]:
a00p[4,1,2]:=a00p[1,4,2]:
a00p[3,2,1]:=a00p[2,3,1]:
a00p[4,2,2]:=a00p[2,4,2]:

b00s:=a00s:

b00p:=array(1..4,1..4,1..2):
for i from 1 to 4 do
for j from 1 to 4 do
for k from 1 to 2 do
b00p[i,j,k]:=-a00p[i,j,k]
od: od: od:

alpha:=(1/2)*arctan(((mgg2^2+MZ2)/(mgg2^2-MZ2))*tan(2*bet)):

Srotate:=matrix(2,2,[cos(alpha),sin(alpha),-sin(alpha),cos(alpha)]):
Protate:=matrix(2,2,[cos(bet),sin(bet),-sin(bet),cos(bet)]):

apps:=array(1..2,1..2,1..2):
for i from 1 to 2 do
for j from 1 to 2 do
for k from 1 to 2 do
apps[i,j,k]:=0
od: od: od:
apps[1,2,1]:=g2/sqrt(2):
apps[2,1,2]:=g2/sqrt(2):

appp:=array(1..2,1..2,1..2):
for i from 1 to 2 do
for j from 1 to 2 do
for k from 1 to 2 do
appp[i,j,k]:=0
od: od: od:
appp[1,2,1]:=g2/sqrt(2):
```

```
appp[2,1,2]:=-g2/sqrt(2):

bpps:=array(1..2,1..2,1..2):
for i from 1 to 2 do
for j from 1 to 2 do
for k from 1 to 2 do
bpps[i,j,k]:=apps[j,i,k]
od: od: od:

bppp:=array(1..2,1..2,1..2):
for i from 1 to 2 do
for j from 1 to 2 do
for k from 1 to 2 do
bppp[i,j,k]:=-appp[j,i,k]
od: od: od:

a0pch:=array(1..4,1..2,1..2):
for i from 1 to 4 do
for j from 1 to 2 do
for k from 1 to 2 do
a0pch[i,j,k]:=0
od: od: od:
a0pch[1,2,1]:=gprime/sqrt(2):
a0pch[2,2,1]:=g2/sqrt(2):
a0pch[3,1,1]:=-g2:

b0pch:=array(1..4,1..2,1..2):
for i from 1 to 4 do
for j from 1 to 2 do
for k from 1 to 2 do
b0pch[i,j,k]:=0
od: od: od:
b0pch[1,2,2]:=gprime/sqrt(2):
b0pch[2,2,2]:=g2/sqrt(2):
b0pch[4,1,2]:=g2:
```

C.1.2 rot

```
toprot:=matrix(4,2,0):
for i from 1 to 4 do
```

```
a0ttvec:=vector(2,0):
for j from 1 to 2 do
a0ttvec[j]:=a0tt[i,j]
od:
a0ttmass:=evalm(topmix&*a0ttvec):
for k from 1 to 2 do
toprot[i,k]:=a0ttmass[k]
od: od:

botrot:=matrix(4,2,0):
for i from 1 to 4 do
a0bbvec:=vector(2,0):
for j from 1 to 2 do
a0bbvec[j]:=a0bb[i,j]
od:
a0bbmass:=evalm(botmix&*a0bbvec):
for k from 1 to 2 do
botrot[i,k]:=a0bbmass[k]
od: od:

taurot:=matrix(4,2,0):
for i from 1 to 4 do
a0tauvec:=vector(2,0):
for j from 1 to 2 do
a0tauvec[j]:=a0tau[i,j]
od:
a0taumass:=evalm(taumix&*a0tauvec):
for k from 1 to 2 do
taurot[i,k]:=a0taumass[k]
od: od:

chdtrot:=matrix(2,2,0):
for i from 1 to 2 do
apdtvec:=vector(2,0):
for j from 1 to 2 do
apdtvec[j]:=apdt[i,j]
od:
apdtmass:=evalm(topmix&*apdtvec):
for k from 1 to 2 do
```

```
chdtrot[i,k]:=apdtmass[k]
od: od:

chbtrot:=matrix(2,2,0):
for i from 1 to 2 do
apbtvec:=vector(2,0):
for j from 1 to 2 do
apbtvec[j]:=apbt[i,j]
od:
apbtmass:=evalm(topmix&*apbtvec):
for k from 1 to 2 do
chbtrot[i,k]:=apbtmass[k]
od: od:

chubrot:=matrix(2,2,0):
for i from 1 to 2 do
apubvec:=vector(2,0):
for j from 1 to 2 do
apubvec[j]:=apub[i,j]
od:
apubmass:=evalm(botmix&*apubvec):
for k from 1 to 2 do
chubrot[i,k]:=apubmass[k]
od: od:

chtbrot:=matrix(2,2):
for i from 1 to 2 do
aptbvec:=vector(2,0):
for j from 1 to 2 do
aptbvec[j]:=aptb[i,j]
od:
aptbmass:=evalm(botmix&*aptbvec):
for k from 1 to 2 do
chtbrot[i,k]:=aptbmass[k]
od: od:

chubrotb:=matrix(2,2,0):
for i from 1 to 2 do
bpubvec:=vector(2,0):
```

```
for j from 1 to 2 do
bpubvec[j]:=bpub[i,j]
od:
bpubmass:=evalm(botmix&*bpubvec):
for k from 1 to 2 do
chubrotb[i,k]:=bpubmass[k]
od: od:

chtbrotb:=matrix(2,2,0):
for i from 1 to 2 do
bptbvec:=vector(2,0):
for j from 1 to 2 do
bptbvec[j]:=bptb[i,j]
od:
bptbmass:=evalm(botmix&*bptbvec):
for k from 1 to 2 do
chtbrotb[i,k]:=bptbmass[k]
od: od:


chdtrotb:=matrix(2,2,0):
for i from 1 to 2 do
bpdtvec:=vector(2,0):
for j from 1 to 2 do
bpdtvec[j]:=bpdt[i,j]
od:
bpdtmass:=evalm(topmix&*bpdtvec):
for k from 1 to 2 do
chdtrotb[i,k]:=bpdtmass[k]
od: od:

chbtrotb:=matrix(2,2,0):
for i from 1 to 2 do
bpbtvec:=vector(2,0):
for j from 1 to 2 do
bpbtvec[j]:=bpbt[i,j]
od:
bpbtmass:=evalm(topmix&*bpbtvec):
for k from 1 to 2 do
```

```
chbtrotb[i,k]:=bpbtmass[k]
od: od:

chnutau:=matrix(2,2,0):
for i from 1 to 2 do
apnutauvec:=vector(2,0):
for j from 1 to 2 do
apnutauvec[j]:=apnutau[i,j]
od:
apnutaumass:=evalm(taumix&*apnutauvec):
for k from 1 to 2 do
chnutau[i,k]:=apnutaumass[k]
od: od:

chnutautau:=matrix(2,2,0):
for i from 1 to 2 do
apnutautauvec:=vector(2,0):
for j from 1 to 2 do
apnutautauvec[j]:=apnutautau[i,j]
od:
apnutautaumass:=evalm(taumix&*apnutautauvec):
for k from 1 to 2 do
chnutautau[i,k]:=apnutautaumass[k]
od: od:

chnutaub:=matrix(2,2,0):
for i from 1 to 2 do
bpnutauvec:=vector(2,0):
for j from 1 to 2 do
bpnutauvec[j]:=bpnutau[i,j]
od:
bpnutaumass:=evalm(taumix&*bpnutauvec):
for k from 1 to 2 do
chnutaub[i,k]:=bpnutaumass[k]
od: od:

chnutautaub:=matrix(2,2,0):
for i from 1 to 2 do
bpnutautauvec:=vector(2,0):
```

```
for j from 1 to 2 do
bpnutautauvec[j]:=bpnutautau[i,j]
od:
bpnutautaumass:=evalm(taumix&*bpnutautauvec):
for k from 1 to 2 do
chnutautaub[i,k]:=bpnutautaumass[k]
od: od:

chtaunutau:=aptaunutau:

chtaunutaub:=bptaunutau:
```

C.1.3 rotb

```
toprotb:=matrix(4,2,0):
for i from 1 to 4 do
b0ttvec:=vector(2,0):
for j from 1 to 2 do
b0ttvec[j]:=b0tt[i,j]
od:
b0ttmass:=evalm(topmix&*b0ttvec):
for k from 1 to 2 do
toprotb[i,k]:=b0ttmass[k]
od: od:

botrotb:=matrix(4,2,0):
for i from 1 to 4 do
b0bbvec:=vector(2,0):
for j from 1 to 2 do
b0bbvec[j]:=b0bb[i,j]
od:
b0bbmass:=evalm(botmix&*b0bbvec):
for k from 1 to 2 do
botrotb[i,k]:=b0bbmass[k]
od: od:

taurotb:=matrix(4,2,0):
for i from 1 to 4 do
b0tauvec:=vector(2,0):
for j from 1 to 2 do
```

```
b0tauvec[j]:=b0tau[i,j]
od:
b0taumass:=evalm(taumix&*b0tauvec):
for k from 1 to 2 do
taurotb[i,k]:=b0taumass[k]
od: od:
```

C.1.4 Wrot

```
pWrot:=matrix(4,2,0):
for i from 1 to 4 do
a0pWvec:=vector(2,0):
for j from 1 to 2 do
a0pWvec[j]:=a0pW[i,j]
od:
a0pWmass:=evalm((conjugate(Vchino))&*a0pWvec):
for k from 1 to 2 do
pWrot[i,k]:=a0pWmass[k]
od: od:

PWrot:=matrix(4,2,0):
for i from 1 to 4 do
a0pWVEC:=vector(2,0):
for j from 1 to 2 do
a0pWVEC[j]:=a0pW[i,j]
od:
a0pWMASS:=evalm(Vchino&*a0pWVEC):
for k from 1 to 2 do
PWrot[i,k]:=a0pWMASS[k]
od: od:

pWrotb:=matrix(4,2,0):
for i from 1 to 4 do
b0pWvec:=vector(2,0):
for j from 1 to 2 do
b0pWvec[j]:=b0pW[i,j]
od:
b0pWmass:=evalm(Uchino&*b0pWvec):
for k from 1 to 2 do
pWrotb[i,k]:=b0pWmass[k]
```

```
od: od:

PWrotb:=matrix(4,2,0):
for i from 1 to 4 do
bOpWVEC:=vector(2,0):
for j from 1 to 2 do
bOpWVEC[j]:=bOpW[i,j]
od:
bOpWMASS:=evalm((conjugate(Uchino))&*bOpWVEC):
for k from 1 to 2 do
PWrotb[i,k]:=bOpWMASS[k]
od: od:

neuWrot:=matrix(4,2,0):
for i from 1 to 2 do
AOpWvec:=vector(4,0):
for j from 1 to 4 do
AOpWvec[j]:=aOpW[j,i]
od:
AOpWmass:=evalm(Nslino&*AOpWvec):
for k from 1 to 4 do
neuWrot[k,i]:=AOpWmass[k]
od: od:

neuWrotb:=matrix(4,2,0):
for i from 1 to 2 do
BOpWvec:=vector(4,0):
for j from 1 to 4 do
BOpWvec[j]:=bOpW[j,i]
od:
BOpWmass:=evalm((conjugate(Nslino))&*BOpWvec):
for k from 1 to 4 do
neuWrotb[k,i]:=BOpWmass[k]
od: od:
```

C.1.5 Zrot

```
NeuZrot:=matrix(4,4,0):
for i from 1 to 4 do
a00Zvec:=vector(4,0):
```

```
for j from 1 to 4 do
a00Zvec[j]:=a00Z[i,j]
od:
a00Zmass:=evalm(Nslino&*a00Zvec):
for k from 1 to 4 do
NeuZrot[i,k]:=a00Zmass[k]
od: od:

NeuZrotb:=matrix(4,4,0):
for i from 1 to 4 do
b00Zvec:=vector(4,0):
for j from 1 to 4 do
b00Zvec[j]:=b00Z[i,j]
od:
b00Zmass:=evalm((conjugate(Nslino))&*b00Zvec):
for k from 1 to 4 do
NeuZrotb[i,k]:=b00Zmass[k]
od: od:

pZrot:=matrix(2,2,0):
for i from 1 to 2 do
appZvec:=vector(2,0):
for j from 1 to 2 do
appZvec[j]:=appZ[i,j]
od:
appZmass:=evalm(Vchino&*appZvec):
for k from 1 to 2 do
pZrot[i,k]:=appZmass[k]
od: od:

pZrotb:=matrix(2,2,0):
for i from 1 to 2 do
bppZvec:=vector(2,0):
for j from 1 to 2 do
bppZvec[j]:=bppZ[i,j]
od:
bppZmass:=evalm((conjugate(Uchino))&*bppZvec):
for k from 1 to 2 do
pZrotb[i,k]:=bppZmass[k]
```

```
od: od:
```

C.1.6 Higgsrot

```
a0pchnew:=array(1..4,1..2,1..2):
for i from 1 to 4 do
for j from 1 to 2 do
newvec:=vector(2,0):
for k from 1 to 2 do
newvec[k]:=a0pch[i,k,j]
od:
newmass:=evalm((conjugate(Uchino))&*newvec):
for l from 1 to 2 do
a0pchnew[i,l,j]:=newmass[l]
od: od: od:

a0pchNEW:=array(1..4,1..2,1..2):
for i from 1 to 2 do
for j from 1 to 2 do
NEWvec:=vector(4,0):
for k from 1 to 4 do
NEWvec[k]:=b0pch[k,i,j]
od:
NEWmass:=evalm((conjugate(Nslino))&*NEWvec):
for l from 1 to 4 do
a0pchNEW[l,i,j]:=NEWmass[l]
od: od: od:

chhiggsrot:=array(1..4,1..2,1..2):
for i from 1 to 4 do
for j from 1 to 2 do
a0pchvec:=vector(2,0):
for k from 1 to 2 do
a0pchvec[k]:=a0pchnew[i,j,k]
od:
a0pchmass:=evalm(Protate&*a0pchvec):
for l from 1 to 2 do
chhiggsrot[i,j,l]:=a0pchmass[l]
od: od: od:
```

```
CHhiggsrot:=array(1..4,1..2,1..2):
for i from 1 to 4 do
for j from 1 to 2 do
a0pchVEC:=vector(2,0):
for k from 1 to 2 do
a0pchVEC[k]:=a0pchNEW[i,j,k]
od:
a0pchMASS:=evalm(Protate&*a0pchVEC):
for l from 1 to 2 do
CHhiggsrot[i,j,l]:=a0pchMASS[l]
od: od: od:

b0pchnew:=array(1..4,1..2,1..2):
for i from 1 to 4 do
for j from 1 to 2 do
vec:=vector(2,0):
for k from 1 to 2 do
vec[k]:=b0pch[i,k,j]
od:
vecmass:=evalm(Vchino&*vec):
for l from 1 to 2 do
b0pchnew[i,l,j]:=vecmass[l]
od: od: od:

b0pchNEW:=array(1..4,1..2,1..2):
for i from 1 to 2 do
for j from 1 to 2 do
VEC:=vector(4,0):
for k from 1 to 4 do
VEC[k]:=a0pch[k,i,j]
od:
VECmass:=evalm(Nslino&*VEC):
for l from 1 to 4 do
b0pchNEW[l,i,j]:=VECmass[l]
od: od: od:

chhiggsrotb:=array(1..4,1..2,1..2):
for i from 1 to 4 do
for j from 1 to 2 do
```

```
b0pchvec:=vector(2,0):
for k from 1 to 2 do
b0pchvec[k]:=b0pchnew[i,j,k]
od:
b0pchmass:=evalm(Protate&*b0pchvec):
for l from 1 to 2 do
chhiggsrotb[i,j,l]:=b0pchmass[l]
od: od: od:

CHhiggsrotb:=array(1..4,1..2,1..2):
for i from 1 to 4 do
for j from 1 to 2 do
b0pchVEC:=vector(2,0):
for k from 1 to 2 do
b0pchVEC[k]:=b0pchNEW[i,j,k]
od:
b0pchMASS:=evalm(Protate&*b0pchVEC):
for l from 1 to 2 do
CHhiggsrotb[i,j,l]:=b0pchMASS[l]
od: od: od:

a00snew:=array(1..4,1..4,1..2):
for i from 1 to 4 do
for j from 1 to 2 do
sikvec:=vector(4,0):
for k from 1 to 4 do
sikvec[k]:=a00s[i,k,j]
od:
sikmass:=evalm((conjugate(Nslino))&*sikvec):
for l from 1 to 4 do
a00snew[i,l,j]:=sikmass[l]
od: od: od:

a00pnew:=array(1..4,1..4,1..2):
for i from 1 to 4 do
for j from 1 to 2 do
Sikvec:=vector(4,0):
for k from 1 to 4 do
Sikvec[k]:=a00p[i,k,j]
```

```
od:
Sikmass:=evalm((conjugate(Nslino))&*Sikvec):
for l from 1 to 4 do
a00pnew[i,l,j]:=Sikmass[l]
od: od: od:

neuhiggsrot:=array(1..4,1..4,1..2):
for i from 1 to 4 do
for j from 1 to 4 do
a00svec:=vector(2,0):
for k from 1 to 2 do
a00svec[k]:=a00snew[i,j,k]
od:
a00smass:=evalm(Srotate&*a00svec):
for l from 1 to 2 do
neuhiggsrot[i,j,l]:=a00smass[l]
od: od: od:

neuhoddrot:=array(1..4,1..4,1..2):
for i from 1 to 4 do
for j from 1 to 4 do
a00pvec:=vector(2,0):
for k from 1 to 2 do
a00pvec[k]:=a00pnew[i,j,k]
od:
a00pmass:=evalm(Protate&*a00pvec):
for l from 1 to 2 do
neuhoddrot[i,j,l]:=a00pmass[l]
od: od: od:

b00snew:=array(1..4,1..4,1..2):
for i from 1 to 4 do
for j from 1 to 2 do
myvec:=vector(4,0):
for k from 1 to 4 do
myvec[k]:=b00s[i,k,j]
od:
mymass:=evalm(Nslino&*myvec):
for l from 1 to 4 do
```

```
b00snew[i,l,j]:=mymass[l]
od: od: od:

b00pnew:=array(1..4,1..4,1..2):
for i from 1 to 4 do
for j from 1 to 2 do
Myvec:=vector(4,0):
for k from 1 to 4 do
Myvec[k]:=b00p[i,k,j]
od:
Mymass:=evalm(Nslino&*Myvec):
for l from 1 to 4 do
b00pnew[i,l,j]:=Mymass[l]
od: od: od:

neuhiggsrotb:=array(1..4,1..4,1..2):
for i from 1 to 4 do
for j from 1 to 4 do
b00svec:=vector(2,0):
for k from 1 to 2 do
b00svec[k]:=b00snew[i,j,k]
od:
b00smass:=evalm(Srotate&*b00svec):
for l from 1 to 2 do
neuhiggsrotb[i,j,l]:=b00smass[l]
od: od: od:

neuhoddrotb:=array(1..4,1..4,1..2):
for i from 1 to 4 do
for j from 1 to 4 do
b00pvec:=vector(2,0):
for k from 1 to 2 do
b00pvec[k]:=b00pnew[i,j,k]
od:
b00pmass:=evalm(Protate&*b00pvec):
for l from 1 to 2 do
neuhoddrotb[i,j,l]:=b00pmass[l]
od: od: od:
```

```
scarf:=array(1..4,1..2,1..2):
for i from 1 to 2 do
for j from 1 to 2 do
scarfvec:=vector(4,0):
for k from 1 to 4 do
scarfvec[k]:=a0pch[k,i,j]
od:
scarfmass:=evalm((conjugate(Nslino))&*scarfvec):
for l from 1 to 4 do
scarf[l,i,j]:=scarfmass[l]
od: od: od:

chhiggsneurot:=array(1..4,1..2,1..2):
for i from 1 to 4 do
for j from 1 to 2 do
A0pchvec:=vector(2,0):
for k from 1 to 2 do
A0pchvec[k]:=scarf[i,j,k]
od:
A0pchmass:=evalm(Protate&*A0pchvec):
for l from 1 to 2 do
chhiggsneurot[i,j,l]:=A0pchmass[l]
od: od: od:

gloves:=array(1..4,1..2,1..2):
for i from 1 to 2 do
for j from 1 to 2 do
glovesvec:=vector(4,0):
for k from 1 to 4 do
glovesvec[k]:=b0pch[k,i,j]
od:
glovesmass:=evalm(Nslino&*glovesvec):
for l from 1 to 4 do
gloves[l,i,j]:=glovesmass[l]
od: od: od:

chhiggsneurotb:=array(1..4,1..2,1..2):
for i from 1 to 4 do
for j from 1 to 2 do
```

```
BOpchvec:=vector(2,0):
for k from 1 to 2 do
BOpchvec[k]:=gloves[i,j,k]
od:
BOpchmass:=evalm(Protate&*BOpchvec):
for l from 1 to 2 do
chhiggsneurotb[i,j,l]:=BOpchmass[l]
od: od: od:

Pops:=array(1..2,1..2,1..2):
for i from 1 to 2 do
for j from 1 to 2 do
Popvec:=vector(2,0):
for k from 1 to 2 do
Popvec[k]:=apps[i,k,j]
od:
Popmass:=evalm((conjugate(Uchino))&*Popvec):
for l from 1 to 2 do
Pops[i,l,j]:=Popmass[l]
od: od: od:

Popp:=array(1..2,1..2,1..2):
for i from 1 to 2 do
for j from 1 to 2 do
Poppvec:=vector(2,0):
for k from 1 to 2 do
Poppvec[k]:=appp[i,k,j]
od:
Poppmass:=evalm((conjugate(Uchino))&*Poppvec):
for l from 1 to 2 do
Popp[i,l,j]:=Poppmass[l]
od: od: od:

chp0H:=array(1..2,1..2,1..2):
for i from 1 to 2 do
for j from 1 to 2 do
Appsvec:=vector(2,0):
for k from 1 to 2 do
Appsvec[k]:=Pops[i,j,k]
```

```
od:
Appsmass:=evalm(Srotate&*Appsvec):
for l from 1 to 2 do
chpOH[i,j,l]:=Appsmass[l]
od: od: od:

chpodd:=array(1..2,1..2,1..2):
for i from 1 to 2 do
for j from 1 to 2 do
Apppvec:=vector(2,0):
for k from 1 to 2 do
Apppvec[k]:=Popp[i,j,k]
od:
Apppmass:=evalm(Protate&*Apppvec):
for l from 1 to 2 do
chpodd[i,j,l]:=Apppmass[l]
od: od: od:

Mops:=array(1..2,1..2,1..2):
for i from 1 to 2 do
for j from 1 to 2 do
Mopvec:=vector(2,0):
for k from 1 to 2 do
Mopvec[k]:=bpps[i,k,j]
od:
Mopmass:=evalm(Vchino&*Mopvec):
for l from 1 to 2 do
Mops[i,l,j]:=Mopmass[l]
od: od: od:

Mopp:=array(1..2,1..2,1..2):
for i from 1 to 2 do
for j from 1 to 2 do
Moppvec:=vector(2,0):
for k from 1 to 2 do
Moppvec[k]:=bppp[i,k,j]
od:
Moppmass:=evalm(Vchino&*Moppvec):
for l from 1 to 2 do
```

```
Mopp[i,l,j]:=Moppmass[l]
od: od: od:

chp0Hb:=array(1..2,1..2,1..2):
for i from 1 to 2 do
for j from 1 to 2 do
Bppsvec:=vector(2,0):
for k from 1 to 2 do
Bppsvec[k]:=Mops[i,j,k]
od:
Bppsmass:=evalm(Srotate&*Bppsvec):
for l from 1 to 2 do
chp0Hb[i,j,l]:=Bppsmass[l]
od: od: od:

chpoddb:=array(1..2,1..2,1..2):
for i from 1 to 2 do
for j from 1 to 2 do
Bpppvec:=vector(2,0):
for k from 1 to 2 do
Bpppvec[k]:=Mopp[i,j,k]
od:
Bpppmass:=evalm(Protate&*Bpppvec):
for l from 1 to 2 do
chpoddb[i,j,l]:=Bpppmass[l]
od: od: od:
```

C.2 neutralino

```
#This calculates the neutralino mass with radiative
#corrections to 1-loop

read "bterms":

NeuLferm:=matrix(4,4,0):
for i from 1 to 4 do
for j from 1 to 4 do
NeuLferm[i,j]:=2*3*a0uu[i,1]*a0uu[j,1]*B1upup1
+3*toprot[i,1]*toprot[j,1]*B1toptop1
+2*3*a0uu[i,2]*a0uu[j,2]*B1upup2
```

```
+3*toprot[i,2]*toprot[j,2]*B1toptop2
+2*3*a0dd[i,1]*a0dd[j,1]*B1downdown1
+3*botrot[i,1]*botrot[j,1]*B1botbot1
+2*3*a0dd[i,2]*a0dd[j,2]*B1downdown2
+3*botrot[i,2]*botrot[j,2]*B1botbot2
+2*a0ee[i,1]*a0ee[j,1]*B1elel1
+taurot[i,1]*taurot[j,1]*B1tautau1
+2*a0ee[i,2]*a0ee[j,2]*B1elel2
+taurot[i,2]*taurot[j,2]*B1tautau2
od: od:

NeuRferm:=matrix(4,4,0):
for i from 1 to 4 do
for j from 1 to 4 do
NeuRferm[i,j]:=2*3*b0uu[i,1]*b0uu[j,1]*B1upup1
+3*toprotb[i,1]*toprotb[j,1]*B1toptop1
+2*3*b0uu[i,2]*b0uu[j,2]*B1upup2
+3*toprotb[i,2]*toprotb[j,2]*B1toptop2
+2*3*b0dd[i,1]*b0dd[j,1]*B1downdown1
+3*botrotb[i,1]*botrotb[j,1]*B1botbot1
+2*3*b0dd[i,2]*b0dd[j,2]*B1downdown2
+3*botrotb[i,2]*botrotb[j,2]*B1botbot2
+2*b0ee[i,1]*b0ee[j,1]*B1elel1
+taurotb[i,1]*taurotb[j,1]*B1tautau1
+2*b0ee[i,2]*b0ee[j,2]*B1elel2
+taurotb[i,2]*taurotb[j,2]*B1tautau2
+2*b0nn[i]*b0nn[j]*B1nunu
+b0nn[i]*b0nn[j]*B1nutau
od: od:


#Need factor of 2 on W part

NeuLW:=matrix(4,4,0):
for i from 1 to 4 do
for j from 1 to 4 do
NeuLW[i,j]:=PWrot[i,1]*PWrot[j,1]*B1ch1W
+PWrot[i,2]*PWrot[j,2]*B1ch2W
od: od:
```

```
NeuRW:=matrix(4,4,0):
for i from 1 to 4 do
for j from 1 to 4 do
NeuRW[i,j]:=PWrotb[i,1]*PWrotb[j,1]*B1ch1W
+PWrotb[i,2]*PWrotb[j,2]*B1ch2W
od: od:


NeuLZ:=matrix(4,4,0):
for i from 1 to 4 do
for j from 1 to 4 do
NeuLZ[i,j]:=Re((conjugate(NeuZrot[i,1]))*NeuZrot[j,1]*B1neut1Z
+(conjugate(NeuZrot[i,2]))*NeuZrot[j,2]*B1neut2Z
+(conjugate(NeuZrot[i,3]))*NeuZrot[j,3]*B1neut3Z
+(conjugate(NeuZrot[i,4]))*NeuZrot[j,4]*B1neut4Z)
od: od:

NeuRZ:=matrix(4,4,0):
for i from 1 to 4 do
for j from 1 to 4 do
NeuRZ[i,j]:=Re((conjugate(NeuZrotb[i,1]))*NeuZrotb[j,1]*B1neut1Z
+(conjugate(NeuZrotb[i,2]))*NeuZrotb[j,2]*B1neut2Z
+(conjugate(NeuZrotb[i,3]))*NeuZrotb[j,3]*B1neut3Z
+(conjugate(NeuZrotb[i,4]))*NeuZrotb[j,4]*B1neut4Z)
od: od:

NeuLchH:=matrix(4,4,0):
for i from 1 to 4 do
for j from 1 to 4 do
NeuLchH[i,j]:=(conjugate(chhiggsrot[i,1,1]))*chhiggsrot[j,1,1]*B1ch1chHv
+(conjugate(chhiggsrot[i,1,2]))*chhiggsrot[j,1,2]*B1ch1chH
+(conjugate(chhiggsrot[i,2,1]))*chhiggsrot[j,2,1]*B1ch2chHv
+(conjugate(chhiggsrot[i,2,2]))*chhiggsrot[j,2,2]*B1ch2chH
od: od:

NeuRchH:=matrix(4,4,0):
for i from 1 to 4 do
for j from 1 to 4 do
```

```
NeuRchH[i,j]:=
(conjugate(chhiggsrotb[i,1,1]))*chhiggsrotb[j,1,1]*B1ch1chHv
+(conjugate(chhiggsrotb[i,1,2]))*chhiggsrotb[j,1,2]*B1ch1chH
+(conjugate(chhiggsrotb[i,2,1]))*chhiggsrotb[j,2,1]*B1ch2chHv
+(conjugate(chhiggsrotb[i,2,2]))*chhiggsrotb[j,2,2]*B1ch2chH
od: od:


#Factor of 1/2 on neutral Higgs loops

NeuLOH;=matrix(4,4,0):
for i from 1 to 4 do
for j from 1 to 4 do
NeuLOH[i,j]:=
Re(conjugate(neuhiggsrot[i,1,1])*neuhiggsrot[j,1,1]*B1neut1OH1
+conjugate(neuhiggsrot[i,1,2])*neuhiggsrot[j,1,2]*B1neut1OH2
+conjugate(neuhiggsrot[i,2,1])*neuhiggsrot[j,2,1]*B1neut2OH1
+conjugate(neuhiggsrot[i,2,2])*neuhiggsrot[j,2,2]*B1neut2OH2
+conjugate(neuhiggsrot[i,3,1])*neuhiggsrot[j,3,1]*B1neut3OH1
+conjugate(neuhiggsrot[i,3,2])*neuhiggsrot[j,3,2]*B1neut3OH2
+conjugate(neuhiggsrot[i,4,1])*neuhiggsrot[j,4,1]*B1neut4OH1
+conjugate(neuhiggsrot[i,4,2])*neuhiggsrot[j,4,2]*B1neut4OH2
+conjugate(neuhoddrot[i,1,1])*neuhoddrot[j,1,1]*B1neut1A
+conjugate(neuhoddrot[i,1,2])*neuhoddrot[j,1,2]*B1neut1Av
+conjugate(neuhoddrot[i,2,1])*neuhoddrot[j,2,1]*B1neut2A
+conjugate(neuhoddrot[i,2,2])*neuhoddrot[j,2,2]*B1neut2Av
+conjugate(neuhoddrot[i,3,1])*neuhoddrot[j,3,1]*B1neut3A
+conjugate(neuhoddrot[i,3,2])*neuhoddrot[j,3,2]*B1neut3Av
+conjugate(neuhoddrot[i,4,1])*neuhoddrot[j,4,1]*B1neut4A
+conjugate(neuhoddrot[i,4,2])*neuhoddrot[j,4,2]*B1neut4Av)
od: od:

NeuROH:=matrix(4,4,0):
for i from 1 to 4 do
for j from 1 to 4 do
NeuROH[i,j]:=
Re(conjugate(neuhiggsrotb[i,1,1])*neuhiggsrotb[j,1,1]*B1neut1OH1
+conjugate(neuhiggsrotb[i,1,2])*neuhiggsrotb[j,1,2]*B1neut1OH2
+conjugate(neuhiggsrotb[i,2,1])*neuhiggsrotb[j,2,1]*B1neut2OH1
```

```
+conjugate(neuhiggsrotb[i,2,2])*neuhiggsrotb[j,2,2]*B1neut20H2
+conjugate(neuhiggsrotb[i,3,1])*neuhiggsrotb[j,3,1]*B1neut30H1
+conjugate(neuhiggsrotb[i,3,2])*neuhiggsrotb[j,3,2]*B1neut30H2
+conjugate(neuhiggsrotb[i,4,1])*neuhiggsrotb[j,4,1]*B1neut40H1
+conjugate(neuhiggsrotb[i,4,2])*neuhiggsrotb[j,4,2]*B1neut40H2
+conjugate(neuhoddrotb[i,1,1])*neuhoddrotb[j,1,1]*B1neut1A
+conjugate(neuhoddrotb[i,1,2])*neuhoddrotb[j,1,2]*B1neut1Av
+conjugate(neuhoddrotb[i,2,1])*neuhoddrotb[j,2,1]*B1neut2A
+conjugate(neuhoddrotb[i,2,2])*neuhoddrotb[j,2,2]*B1neut2Av
+conjugate(neuhoddrotb[i,3,1])*neuhoddrotb[j,3,1]*B1neut3A
+conjugate(neuhoddrotb[i,3,2])*neuhoddrotb[j,3,2]*B1neut3Av
+conjugate(neuhoddrotb[i,4,1])*neuhoddrotb[j,4,1]*B1neut4A
+conjugate(neuhoddrotb[i,4,2])*neuhoddrotb[j,4,2]*B1neut4Av   )
od: od:


NeuLfull:=matrix(4,4,0):
for i from 1 to 4 do
for j from 1 to 4 do
NeuLfull[i,j]:=
evalf(1/(16*Pi^2))*(NeuLferm[i,j]+2*NeuLW[i,j]+NeuLZ[i,j]
+NeuLchH[i,j]+(1/2)*NeuLOH[i,j])
od: od:

NeuRfull:=matrix(4,4,0):
for i from 1 to 4 do
for j from 1 to 4 do
NeuRfull[i,j]:=
evalf(1/(16*Pi^2))*(NeuRferm[i,j]+2*NeuRW[i,j]+NeuRZ[i,j]
+NeuRchH[i,j]+(1/2)*NeuROH[i,j])
od: od:


#Note factor of 2

NeuSferm:=matrix(4,4,0):
for i from 1 to 4 do
for j from 1 to 4 do
NeuSferm[i,j]:=3*toprotb[i,1]*toprot[j,1]*mtopspec*B0toptop1
```

```
+3*toprotb[i,2]*toprot[j,2]*mtopspec*B0toptop2
+3*botrotb[i,1]*botrot[j,1]*mbotspec*B0botbot1
+3*botrotb[i,2]*botrot[j,2]*mbotspec*B0botbot2
+taurotb[i,1]*taurot[j,1]*mtaulepspec*B0tautau1
+taurotb[i,2]*taurot[j,2]*mtaulepspec*B0tautau2
od: od:


#W contribution has factor of (-8)

NeuSW:=matrix(4,4,0):
for i from 1 to 4 do
for j from 1 to 4 do
NeuSW[i,j]:=PWrotb[i,1]*PWrot[j,1]*chino1*B0ch1W
+PWrotb[i,2]*PWrot[j,2]*chino2*B0ch2W
od: od:


#Z contribution has factor of (-4)

NeuSZ:=matrix(4,4,0):
for i from 1 to 4 do
for j from 1 to 4 do
NeuSZ[i,j]:=
Re((conjugate(NeuZrotb[i,1]))*NeuZrot[j,1]*slino1*B0neut1Z
+(conjugate(NeuZrotb[i,2]))*NeuZrot[j,2]*slino2*B0neut2Z
+(conjugate(NeuZrotb[i,3]))*NeuZrot[j,3]*slino3*B0neut3Z
+(conjugate(NeuZrotb[i,4]))*NeuZrot[j,4]*slino4*B0neut4Z)
od: od:


#Charged Higgs contribution has factor 2

NeuSchH:=matrix(4,4,0):
for i from 1 to 4 do
for j from 1 to 4 do
NeuSchH[i,j]:=
(conjugate(chhiggsrotb[i,1,1]))*chhiggsrot[j,1,1]*chino1*B0ch1chHv
+(conjugate(chhiggsrotb[i,1,2]))*chhiggsrot[j,1,2]*chino1*B0ch1chH
```

```
+(conjugate(chhiggsrotb[i,2,1]))*chhiggsrot[j,2,1]*chino2*B0ch2chHv
+(conjugate(chhiggsrotb[i,2,2]))*chhiggsrot[j,2,2]*chino2*B0ch2chH
od: od:


#Neutral Higgs contribution

NeuSOH:=matrix(4,4,0):
for i from 1 to 4 do
for j from 1 to 4 do
NeuSOH[i,j]:=
Re(conjugate(neuhiggsrotb[i,1,1])*neuhiggsrot[j,1,1]*slino1*B0neut10H1
+conjugate(neuhiggsrotb[i,1,2])*neuhiggsrot[j,1,2]*slino1*B0neut10H2
+conjugate(neuhiggsrotb[i,2,1])*neuhiggsrot[j,2,1]*slino2*B0neut20H1
+conjugate(neuhiggsrotb[i,2,2])*neuhiggsrot[j,2,2]*slino2*B0neut20H2
+conjugate(neuhiggsrotb[i,3,1])*neuhiggsrot[j,3,1]*slino3*B0neut30H1
+conjugate(neuhiggsrotb[i,3,2])*neuhiggsrot[j,3,2]*slino3*B0neut30H2
+conjugate(neuhiggsrotb[i,4,1])*neuhiggsrot[j,4,1]*slino4*B0neut40H1
+conjugate(neuhiggsrotb[i,4,2])*neuhiggsrot[j,4,2]*slino4*B0neut40H2
+conjugate(neuhoddrotb[i,1,1])*neuhoddrot[j,1,1]*slino1*B0neut1Av
+conjugate(neuhoddrotb[i,1,2])*neuhoddrot[j,1,2]*slino1*B0neut1A
+conjugate(neuhoddrotb[i,2,1])*neuhoddrot[j,2,1]*slino2*B0neut2Av
+conjugate(neuhoddrotb[i,2,2])*neuhoddrot[j,2,2]*slino2*B0neut2A
+conjugate(neuhoddrotb[i,3,1])*neuhoddrot[j,3,1]*slino3*B0neut3Av
+conjugate(neuhoddrotb[i,3,2])*neuhoddrot[j,3,2]*slino3*B0neut3A
+conjugate(neuhoddrotb[i,4,1])*neuhoddrot[j,4,1]*slino4*B0neut4Av
+conjugate(neuhoddrotb[i,4,2])*neuhoddrot[j,4,2]*slino4*B0neut4A)
od: od:


NeuSfull:=matrix(4,4,0):
for i from 1 to 4 do
for j from 1 to 4 do
NeuSfull[i,j]:=
evalf(1/(16*Pi^2))*(2*NeuSferm[i,j]-8*NeuSW[i,j]-4*NeuSZ[i,j]
+2*NeuSchH[i,j]+NeuSOH[i,j])
od: od:
```

```
Leftbit:=evalm(mslino&*NeuLfull):
Rightbit:=evalm(NeuRfull&*mslino):

deltaslino:=(-1)*evalm(Rightbit+Leftbit+NeuSfull):

#One Loop Neutralino Mass Matrix


NeutMasses:=evalm(mslino+0.5*deltaslino
+0.5*(transpose(deltaslino))):

#Diagonal mass matrix with all positive eigenvalues
evalf(Svd(NeutMasses,Uneut,Vneut)):
Neutdiagcheck:=evalm(transpose(Uneut)&*NeutMasses&*Uneut):
if evalf(Neutdiagcheck[1,1])<0
then NXE1:=I else NXE1:=1 fi:
if evalf(Neutdiagcheck[2,2])<0
then NXE2:=I else NXE2:=1 fi:
if evalf(Neutdiagcheck[3,3])<0
then NXE3:=I else NXE3:=1 fi:
if evalf(Neutdiagcheck[4,4])<0
then NXE4:=I else NXE4:=1 fi:
NXEtra:=
matrix(4,4,[NXE1,0,0,0,0,NXE2,0,0,0,0,NXE3,0,0,0,0,NXE4]):
Uneutx:=evalm(Uneut&*NXEtra):
Neutdiag:=evalm(transpose(Uneutx)&*NeutMasses&*Uneutx):


#These are the Neutralino masses (to 1 loop)

Neutralino1:=evalf(Neutdiag[1,1]);
Neutralino2:=evalf(Neutdiag[2,2]);
Neutralino3:=evalf(Neutdiag[3,3]);
Neutralino4:=evalf(Neutdiag[4,4]);
```

C.2.1 bterms

```
#B1-terms


#Quark-squark
```

```
B1upup1:=Re(B1(mspec^2,0,mup1)):
B1upup2:=Re(B1(mspec^2,0,mup2)):
B1toptop1:=Re(B1(mspec^2,mtopspec^2,stop1)):
B1toptop2:=Re(B1(mspec^2,mtopspec^2,stop2)):
B1downdown1:=Re(B1(mspec^2,0,md1)):
B1downdown2:=Re(B1(mspec^2,0,md2)):
B1botbot1:=Re(B1(mspec^2,mbotspec^2,sbot1)):
B1botbot2:=Re(B1(mspec^2,mbotspec^2,sbot2)):

#Lepton-slepton
B1elel1:=Re(B1(mspec^2,0,smu1)):
B1elel2:=Re(B1(mspec^2,0,smu2)):
B1tautau1:=Re(B1(mspec^2,mtaulepspec^2,stau1)):
B1tautau2:=Re(B1(mspec^2,mtaulepspec^2,stau2)):
B1nunu:=Re(B1(mspec^2,0,snumu)):
B1nutau:=Re(B1(mspec^2,0,snutau)):

#Chargino-W
B1ch1W:=Re(B1(mspec^2,chino1^2,MW2)):
B1ch2W:=Re(B1(mspec^2,chino2^2,MW2)):

#Neutralino-Z
B1neut1Z:=Re(B1(mspec^2,slino1^2,MZ2)):
B1neut2Z:=Re(B1(mspec^2,slino2^2,MZ2)):
B1neut3Z:=Re(B1(mspec^2,slino3^2,MZ2)):
B1neut4Z:=Re(B1(mspec^2,slino4^2,MZ2)):

#Chargino-Charged Higgs
B1ch1chH:=Re(B1(mspec^2,chino1^2,mhplus^2)):
B1ch2chH:=Re(B1(mspec^2,chino2^2,mhplus^2)):
B1ch1chHv:=Re(B1(mspec^2,chino1^2,MW2)):
B1ch2chHv:=Re(B1(mspec^2,chino2^2,MW2)):

#Neutralino-Neutral Higgs
B1neut10H2:=Re(B1(mspec^2,slino1^2,mhiggs1^2)):
B1neut10H1:=Re(B1(mspec^2,slino1^2,mhiggs2^2)):
B1neut20H2:=Re(B1(mspec^2,slino2^2,mhiggs1^2)):
B1neut20H1:=Re(B1(mspec^2,slino2^2,mhiggs2^2)):
B1neut30H2:=Re(B1(mspec^2,slino3^2,mhiggs1^2)):
```

```
B1neut3OH1:=Re(B1(mspec^2,slino3^2,mhiggs2^2)):
B1neut4OH2:=Re(B1(mspec^2,slino4^2,mhiggs1^2)):
B1neut4OH1:=Re(B1(mspec^2,slino4^2,mhiggs2^2)):
B1neut1Av:=Re(B1(mspec^2,slino1^2,mA0^2)):
B1neut2Av:=Re(B1(mspec^2,slino2^2,mA0^2)):
B1neut3Av:=Re(B1(mspec^2,slino3^2,mA0^2)):
B1neut4Av:=Re(B1(mspec^2,slino4^2,mA0^2)):
B1neut1A:=Re(B1(mspec^2,slino1^2,MZ2)):
B1neut2A:=Re(B1(mspec^2,slino2^2,MZ2)):
B1neut3A:=Re(B1(mspec^2,slino3^2,MZ2)):
B1neut4A:=Re(B1(mspec^2,slino4^2,MZ2)):


#B0-terms

#Quark-squark
B0toptop1:=Re(B0(mspec^2,mtopspec^2,stop1)):
B0toptop2:=Re(B0(mspec^2,mtopspec^2,stop2)):
B0botbot1:=Re(B0(mspec^2,mbotspec^2,sbot1)):
B0botbot2:=Re(B0(mspec^2,mbotspec^2,sbot2)):

#Lepton-slepton
B0tautau1:=Re(B0(mspec^2,mtaulepspec^2,stau1)):
B0tautau2:=Re(B0(mspec^2,mtaulepspec^2,stau2)):

#Chargino-W
B0ch1W:=Re(B0(mspec^2,chino1^2,MW2)):
B0ch2W:=Re(B0(mspec^2,chino2^2,MW2)):

#Neutralino-Z
B0neut1Z:=Re(B0(mspec^2,slino1^2,MZ2)):
B0neut2Z:=Re(B0(mspec^2,slino2^2,MZ2)):
B0neut3Z:=Re(B0(mspec^2,slino3^2,MZ2)):
B0neut4Z:=Re(B0(mspec^2,slino4^2,MZ2)):

#Chargino-Charged Higgs
B0ch1chH:=Re(B0(mspec^2,chino1^2,mhplus^2)):
B0ch2chH:=Re(B0(mspec^2,chino2^2,mhplus^2)):
B0ch1chHv:=Re(B0(mspec^2,chino1^2,MW2)):
```

```
B0ch2chHv:=Re(B0(mspec^2,chino2^2,MW2)):

#Neutralino-Neutral Higgs
B0neut1OH2:=Re(B0(mspec^2,slino1^2,mhiggs1^2)):
B0neut1OH1:=Re(B0(mspec^2,slino1^2,mhiggs2^2)):
B0neut2OH2:=Re(B0(mspec^2,slino2^2,mhiggs1^2)):
B0neut2OH1:=Re(B0(mspec^2,slino2^2,mhiggs2^2)):
B0neut3OH2:=Re(B0(mspec^2,slino3^2,mhiggs1^2)):
B0neut3OH1:=Re(B0(mspec^2,slino3^2,mhiggs2^2)):
B0neut4OH2:=Re(B0(mspec^2,slino4^2,mhiggs1^2)):
B0neut4OH1:=Re(B0(mspec^2,slino4^2,mhiggs2^2)):
B0neut1A:=Re(B0(mspec^2,slino1^2,mA0^2)):
B0neut2A:=Re(B0(mspec^2,slino2^2,mA0^2)):
B0neut3A:=Re(B0(mspec^2,slino3^2,mA0^2)):
B0neut4A:=Re(B0(mspec^2,slino4^2,mA0^2)):
B0neut1Av:=Re(B0(mspec^2,slino1^2,MZ2)):
B0neut2Av:=Re(B0(mspec^2,slino2^2,MZ2)):
B0neut3Av:=Re(B0(mspec^2,slino3^2,MZ2)):
B0neut4Av:=Re(B0(mspec^2,slino4^2,MZ2)):
```

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